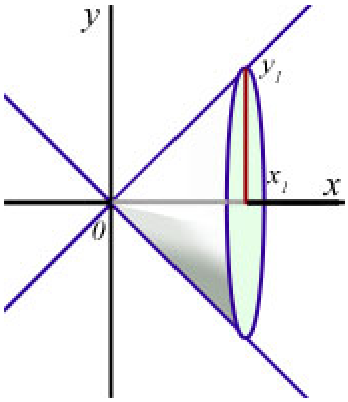
Volumes of Revolution

Method



A volume(rotated around the x-axis)is calculated by first considering a particular value of a function, y_1, up from a value of x at x_1 . The line x_1y_1 may be considered as the 'radius' of the solid at that particular value of x.

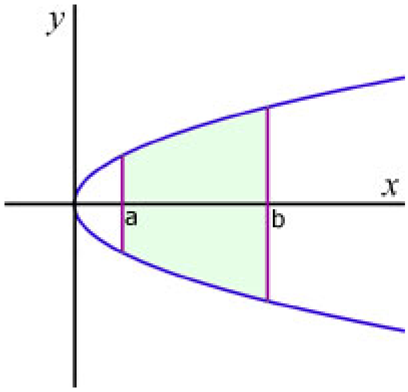
If you were to square the y-value and multiply it by pi, then a cross-sectional area would be created.

Making a solid of revolution is simply the method of summing all the cross-sectional areas along the x-axis between two values of x.

(compare: area of a cylinder = cross-sectional area x length)

The method for solids rotated around the y-axis is similar.

Rotation around the x-axis

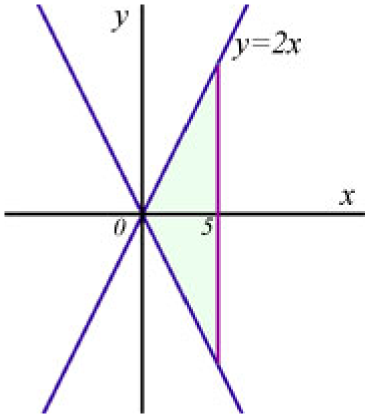


The volume V_x of a curve y=f(x) rotated around the x-axis between the values of x of a and b, is given by:

$$V_x = \pi \int_a^b y^2 dx$$

Example

What is the volume V of the cone swept out by the line y=2x rotated about the x-axis between x=0 and x=5?



using $V_x = \pi \int_a^b y^2 dx$

and $y = 2x$

$\Rightarrow \quad V_{0,5} = \pi \int_0^5 (2x)^2 dx$

$= \pi \int_0^5 4x^2 dx$

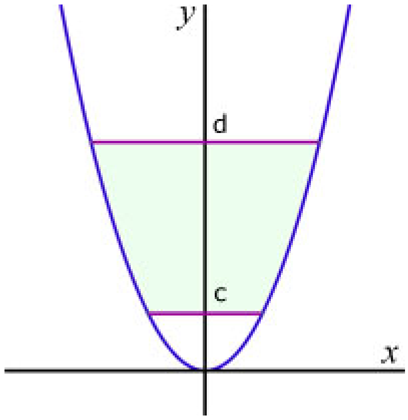
$= \pi \left[\frac{4}{3}(x^3) \right]_0^5$

$= \pi \left[\frac{4}{3}(5^3) \right] - \pi [0]$

$= \pi \frac{4(125)}{3} = \pi \frac{500}{3} = 166.\dot{6}\,\pi$

volume V of cone is $166.\dot{6}\,\pi$

Rotation around the y-axis

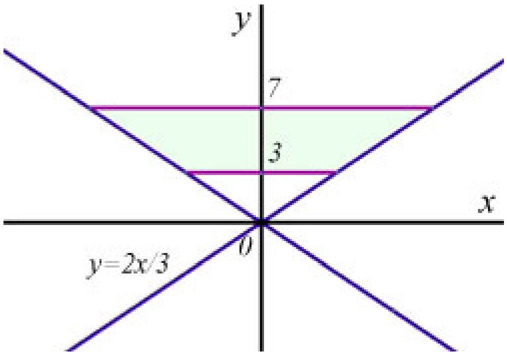


The volume V_y of a curve y=f(x) rotated around the x-axis between the values of y of c and d, is given by:

$$V_y = \pi \int_c^d x^2 dy$$

Example

What is the volume V of the 'frustrum'(cone with smaller cone-shape removed) produced when the line y=2x/3 is rotated around the y-axis, when the centres of the upper and lower areas of the frustrum are at 0,7 and 0,3



using $V_y = \pi\int_c^d x^2 dy$

and $y = \dfrac{2x}{3}, \quad \therefore \quad x = \dfrac{3y}{2}$

$$\Rightarrow \quad V_{3,7} = \pi\int_3^7 \left(\frac{3y}{2}\right)^2 dy$$

$$= \pi\int_3^7 \frac{9}{4}y^2 dy$$

$$= \pi\left[\frac{9}{4}\frac{y^3}{3}\right]_3^7$$

$$= \pi\left[\frac{9}{4}\frac{7^3}{3}\right] - \pi\left[\frac{9}{4}\frac{3^3}{3}\right]$$

$$= \pi\left[\frac{3}{4}(343)\right] - \pi\left[\frac{3}{4}(27)\right]$$

$$= \pi\left[\frac{1029}{4}\right] - \pi\left[\frac{81}{4}\right]$$

$$= \frac{948}{4}\pi = 237\pi$$

volume of cone is 237π