Vectors : General Properties

Notation

A non-zero vector is has the magnitude of a positive real number and a direction in space.

A vector may be represented by two letters describing a line.The order of the letters indicates the direction and the length of the line its magnitude.

$$\text{vector } \overrightarrow{AB} \qquad \text{magnitude } \left|\overrightarrow{AB}\right|$$

An alternative to this notation is to use a single bold letter, for example **C**. Then the magnitude is |**C**| or C.

The Unit Vector

A unit vector eg **a** , has a magnitude of one |**a**|=1 and can point in any direction.

Sometimes a unit vector is written with an accent over it **â**.

Different unit vectors point in different directions.

Hence, if **F** is an ordinary vector,

$$\frac{F}{|F|} = \hat{F}$$

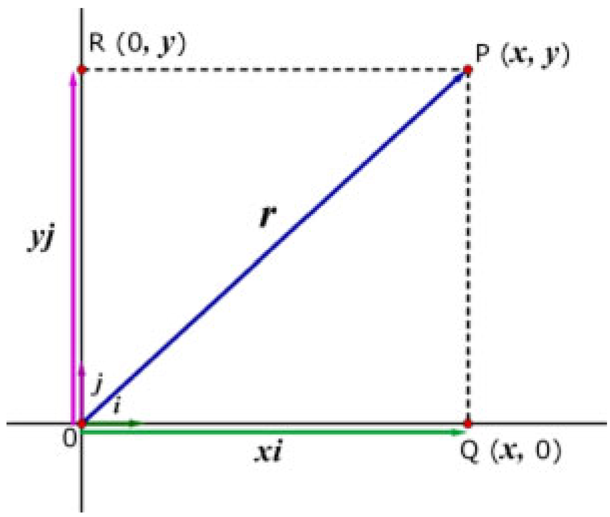
a unit vector in the direction of **F** (^ circumflex accent)

2D representation

P is a point in the x-y plane with coodinates (x,y). $\boldsymbol{i}$ is the unit vector along the x-axis and $\boldsymbol{j}$ is the unit vector along the y-axis.

With Q at $(x,0)$ and R at $(0, y)$:

$$\therefore \quad \overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}$$
$$= \overrightarrow{OQ} + \overrightarrow{OR}$$
$$\Rightarrow \quad \mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

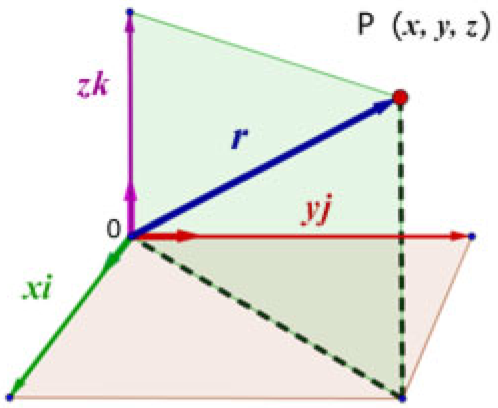


3D representation

P is a point in x-y-z space with coodinates (x, y, z). $\boldsymbol{i}$ is the unit vector along the x-axis, $\boldsymbol{j}$ is the unit vector along the y-axis and $\boldsymbol{k}$ is the unit vector along the z-axis.

$$\overrightarrow{OP} = r = xi + yj + zk$$

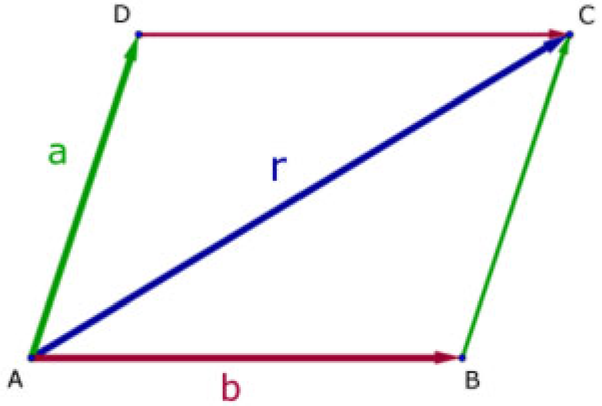
$$\text{length OP} = \left|\overrightarrow{OP}\right| = \sqrt{(x^2 + y^2 + z^2)}$$



Addition(Sum) of vectors

This is also called the Parallelogram or Triangle Law.

If two vectors(**a** & **b**) are represented in magnitude and direction by the adjacent sides of a **parallelogram** from a point, then their resultant(**r**) is represented in magnitude and direction by the diagonal of the parallelogram passing through the point.

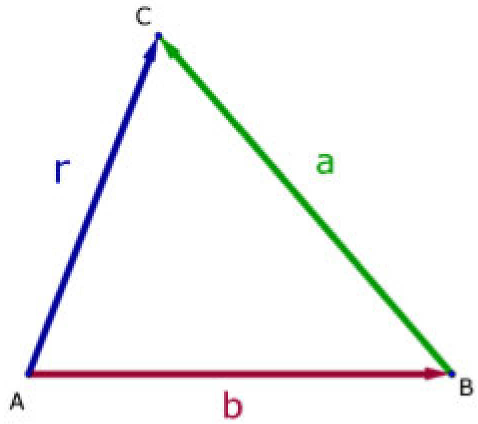


$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$$

$$a = \overrightarrow{AB} \quad b = \overrightarrow{AD}$$

$$a + b = r$$

If two vectors are represented in magnitude and direction by the adjacent sides of a **triangle**, taken in order, then their resultant is represented in magnitude but opposite in direction by the third side.



$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$b = \overrightarrow{AB} \quad a = \overrightarrow{BC}$$

$$b + a = r$$

Scalar Multiplication

Multiplying a vector by a scalar quantity changes its magnitude but not its direction.