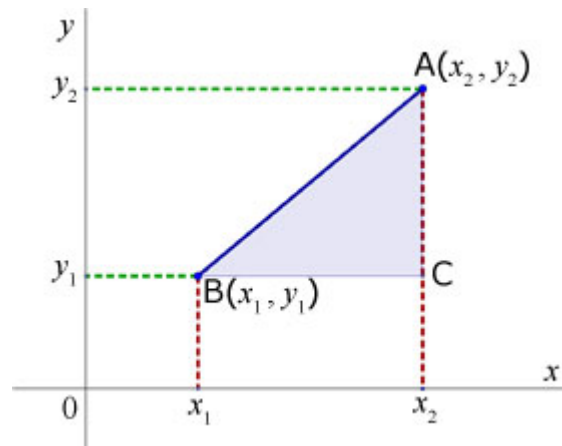


## Line between two points

### Distance of a line between two points



Triangle ABC is a right angled triangle.

therefore

$$(AB)^2 = (BC)^2 + (AC)^2$$

hence

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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Example

What is the distance between points A(5,6) and B(-4,-3) correct to 2 d.p.?

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(x_1, y_1) \equiv (-4, -3)$$

$$\therefore x_1 = -4 \quad y_1 = -3$$

$$(x_2, y_2) \equiv (5, 6)$$

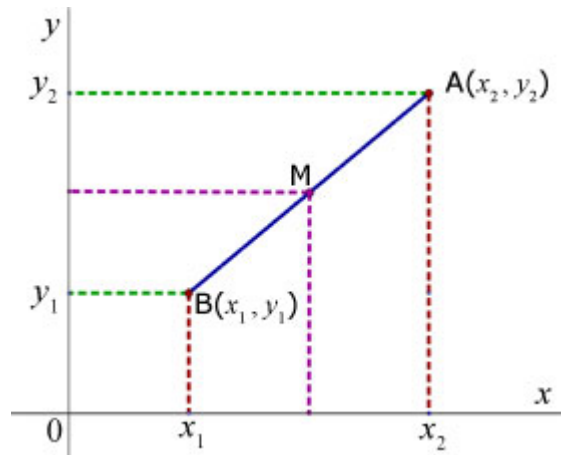
$$\therefore x_2 = 5 \quad y_2 = 6$$

$$\begin{aligned} \Rightarrow AB &= \sqrt{(5 - (-4))^2 + (6 - (-3))^2} \\ &= \sqrt{(5 + 4)^2 + (6 + 3)^2} \\ &= \sqrt{(9)^2 + (9)^2} \\ &= \sqrt{81 + 81} \\ &= \sqrt{162} \\ &= 12.728 \end{aligned}$$

$$\underline{AB = 12.73} \quad (2 \text{ d.p.})$$

Finding the mid-point of a line between two points

The x-y coordinates of the midpoint M between two point A, B is found by taking the average of the x-coordinates( $x_1$  ,  $x_2$ ), then repeating for the y-coordinates( $y_1$  ,  $y_2$ )



the coordinates of M are  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example

What are the coordinates of the mid-point of the line joining the coordinates (4,7) and (-8,8)?

$$(x_1, y_1) \equiv (4, 7) \quad x_1 = 4, \quad y_1 = 7$$

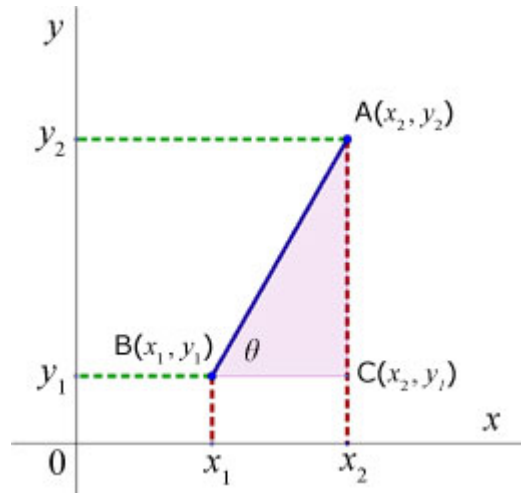
$$(x_2, y_2) \equiv (-8, 8) \quad x_2 = -8, \quad y_2 = 8$$

mid-point coordinates  $(x_M, y_M)$  are given by

$$x_M = \frac{x_1 + x_2}{2} = \frac{4 + (-8)}{2} = \frac{4 - 8}{2} = \frac{-4}{2} = -2$$

$$y_M = \frac{y_1 + y_2}{2} = \frac{7 + 8}{2} = \frac{15}{2} = 7.5$$

the coordinates of the mid-point are (-2, 7.5)

Finding the gradient of a line

the gradient( $m$ ) of line AB is given by:

$$\begin{aligned}\text{gradient}(m) &= \frac{y\text{-step}}{x\text{-step}} \\ &= \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$

note - the gradient  $m$  is equal to  $\tan \theta$

$$m = \tan \theta$$

Example

To 2 decimal places, what is the gradient of the line joining the coordinates (-5,6) and (9,-7)?

$$\begin{aligned}(x_1, y_1) &\equiv (-5, 6) & x_1 &= -5, & y_1 &= 6 \\(x_2, y_2) &\equiv (9, -7) & x_2 &= 9, & y_2 &= -7\end{aligned}$$

$$\begin{aligned}\text{gradient}(m) &= \frac{y_2 - y_1}{x_2 - x_1} \\&= \frac{(-7) - 6}{9 - (-5)} \\&= \frac{-7 - 6}{9 + 5} \\&= \frac{-13}{14} \\&= -0.92857\end{aligned}$$

gradient(m) is -0.93 (2 d.p.)