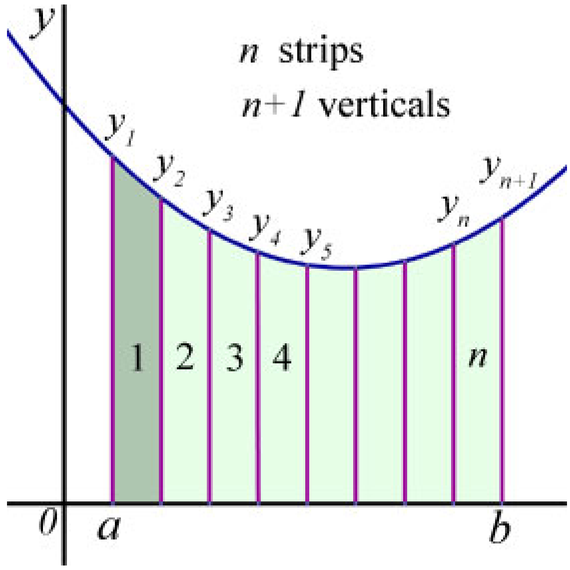
The Trapezium Rule

Theory & method

The Trapezium Rule is a method of finding the approximate value of an integral between two limits.

The area involved is divided up into a number of parallel strips of equal width.



Each area is considered to be a trapezium(trapezoid).

If there are **n** vertical strips then there are **n+1** vertical lines(ordinates) bounding them.

The limits of the integral are between **a** and **b**, and each vertical line has length y_1 y_2 y_3... y_{n+1}

$$\text{width of each strip} = \frac{(b-a)}{n}$$

$$\text{area of first strip}(n=1)$$
$$= (\text{width of strip}) \times (\text{average length of verticals})$$
$$= \frac{(b-a)}{n}\left(\frac{y_2+y_1}{2}\right)$$

Therefore in terms of the all the vertical strips, the integral is given by:

$$\int_a^b f(x)dx$$
$$\approx\left(\frac{b-a}{n}\right)\left[\frac{1}{2}(y_1+y_2)+\frac{1}{2}(y_2+y_3)+\frac{1}{2}(y_3+y_4)+..\frac{1}{2}(y_n+y_{n+1})\right]$$
$$\approx\left(\frac{b-a}{n}\right)\left[\frac{1}{2}(y_1+y_{n+1})+(y_2+y_3+y_4+\ldots+y_n)\right]$$

approx. integral = (strip width) x (average of first and last y-values, plus the sum of all **y** values between the second and second-last value)

Example #1

using a strip width of $\frac{\pi}{10}$, evaluate $\int_0^{\frac{2\pi}{5}} \tan x dx$

(answer to 2 d.p.)

$y_1 = \tan(0) = \quad 0$

$y_2 = \tan\left(\frac{\pi}{10}\right) = \quad 0.3249$

$y_3 = \tan\left(\frac{2\pi}{10}\right) = \quad 0.7265$

$y_4 = \tan\left(\frac{3\pi}{10}\right) = \quad \underline{1.3760}$

2.4274

$y_5 = \tan\left(\frac{4\pi}{10}\right) = \quad \underline{3.078}$

3.078

$$\int_0^{\frac{2\pi}{5}} \tan x dx = \frac{\pi}{10}\left[\frac{3.078}{2} + 2.4274\right]$$

$$= \frac{\pi}{10}\left[1.539 + 2.4274\right]$$

$$= 0.3142 \times 3.966 = 4.594$$

$$\int_0^{\frac{2\pi}{5}} \tan x dx \approx 4.59 \quad (2 \text{ d.p.})$$

Example #2

evaluate $\int_3^8 x^2 dx$ using strips of width '1' unit.

x		y	y
3	y_1	9	
4	y_2		16
5	y_3		25
6	y_4		36
7	y_5		49
8	y_6	64	
		73	126

$$\int_3^8 x^2 dx = 1\left[\frac{73}{2} + 126\right]$$

$$= 36.5 + 126$$

$$= \underline{162.5}$$