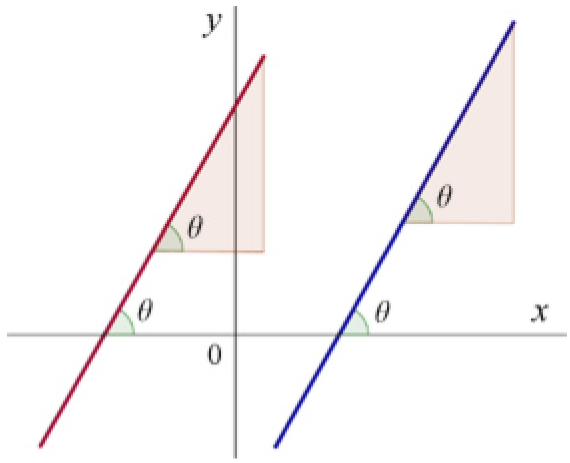
more on straight lines

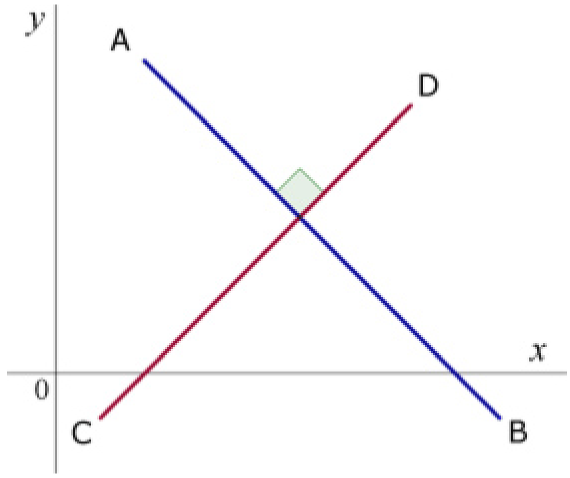
Parallel lines



Parallel lines make equal corresponding angles(θ) with the x-axis.

Therefore their gradients are equal.

Perpendicular lines

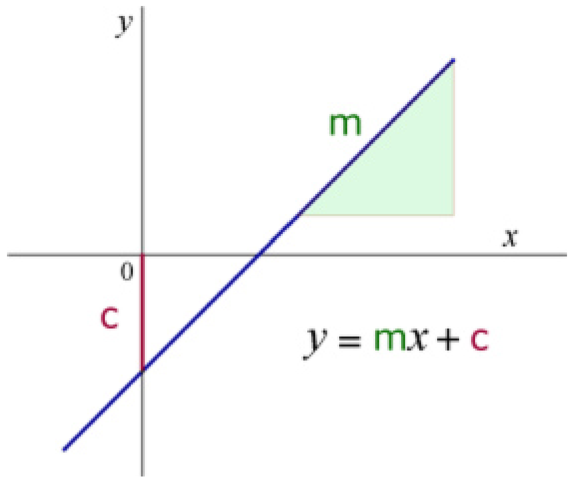


If two lines are perpendicular to eachother, the product of their gradients is -1.

If the gradient of AB is m_1 and the gradient of CD is m_2 , then:

$$m_1 m_2 = -1 \quad \text{or} \quad m_1 = -\frac{1}{m_2}$$

Equation of a straight line y = mx + c



The equation of a straight line is given by:

$$y = mx + c$$

- ***m*** is the gradient of the line
- ***c*** is the intercept on the y-axis

Example

What is the equation of the straight line with gradient 3 that crosses the y-axis at y= -3 ?

$$m = 3,\ c = -3 \ \therefore \text{ the equation is } \ y = 3x - 3$$

Finding the intersection point between two straight lines

There are two types of problem here. One where the lines are not perpendicular to eachother and the other when they are.

To solve the former all that is needed is to solve the equations of the lines simultaneously.

With the later, only one equation is given and the second equation must be worked out from the information supplied. then it is a matter of proceding as before ie to solve the two equations simultaneously.

Example #1

Find the intersection point of the two straight lines:

$$y = 3x + 4 \qquad (i$$
$$y = x + 3 \qquad (ii$$

multiply (ii by 3, subtract (ii from (i

$$y = 3x + 4$$
$$-\underline{(3y = 3x + 9)}$$
$$-2y = \quad -5$$
$$\therefore \underline{y = 2.5}$$

substituting for y in equation (ii

$$y = x + 3$$
$$2.5 = x + 3$$
$$x = 2.5 - 3$$
$$\therefore \underline{x = -0.5}$$

$$\underline{\text{the point of intersection is } (-0.5, 2.5)}$$

Example #2

A straight line $y = 2x + 4.5$ intersects another perpendicularly. If the second straight line has an intercept of -0.5 on the y-axis, what are the coodinates of the point of intersection of the two lines? (answer to 1 d.p.)

because the lines are perpendicular,
gradient m of second line is given by

$$(m)(2) = -1$$

$$\therefore \quad m = -\frac{1}{2}$$

since the intercept of the 2nd. equation is -0.5
using the form $y = mx + c$ its equation is:

$$y = -0.5x - 0.5$$

treating the two equations simultaneously

$$y = 2x + 4.5 \qquad (i$$

$$y = -0.5x - 0.5 \qquad (ii$$

multiplying (ii by 4, add (ii & (i

$$y = 2x + 4.5$$

$$y = -2x - 2$$

$$4y = 2.5$$

$$\underline{y = 0.625}$$

substituting for y in equation (i

$$y = 2x + 4.5$$

$$0.625 = 2x + 4.5$$

$$2x = 0.625 - 4.5$$

$$x = \frac{-3.875}{2}$$

$$\underline{x = -1.938}$$

the point of intersection is $(-1.9, 0.6)$

Finding the eq. of a straight line from one point + gradient

Solution is by using the expression for gradient(m) for an actual point(x_1,y_1) and a generalized point(x,y).

$$\text{gradient}(m) = \frac{y-\text{step}}{x-\text{step}} = \frac{y-y_1}{x-x_1}$$

The straight line equation is found by substituting values of x_1, y_1 and m into the above.

Example

A line of gradient 3 passes through a point (2,5). What is the equation of the line?

$$m = 3$$
$$x_1 = 2, \quad y_1 = 5$$

$$\text{using} \qquad m = \frac{y-y_1}{x-x_1}$$
$$\Rightarrow \qquad 3 = \frac{y-5}{x-2}$$
$$3(x-2) = y-5$$
$$3x-6 = y-5$$
$$3x-1 = y$$

$$\underline{y = 3x-1}$$

Finding the equation of a straight line from two points

Solution is by first finding the gradient *m* from the x and y values from the points (x_1, y_1) and (x_2, y_2)

$$\text{gradient}(m) = \frac{y-\text{step}}{x-\text{step}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Then we use the expression again, but this time with one actual point and a generalized point(x,y).

$$\text{gradient}(m) = \frac{y-\text{step}}{x-\text{step}} = \frac{y - y_1}{x - x_1}$$

The straight line equation is found by substituting for x_1, y_1 and m.

Example

Find the equation of the line between the two points (2,3) and (-5,7).

$$x_1 = 2, \quad y_1 = 3$$
$$x_2 = -5, \quad y_2 = 7$$

using $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{7-3}{-5-2}$$

$$m = -\frac{4}{7}$$

using $\quad m = \dfrac{y - y_1}{x - x_1}$

substituting for x_1 and y_1

$$-\frac{4}{7} = \frac{y-3}{x-2}$$
$$-4(x-2) = 7(y-3)$$
$$-4x + 8 = 7y - 21$$
$$-4x + 29 = 7y$$
$$y = -\tfrac{4}{7}x + \tfrac{29}{7}$$

$$\underline{y = -\tfrac{4}{7}x + 4\tfrac{1}{7}}$$