Sine, Cosine, Tangent

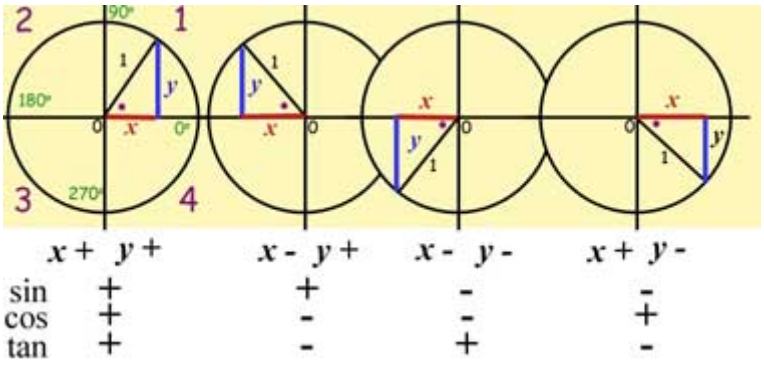
The General Angle

Consider a radius of length '1' rotating anti-clockwise about the origin. The coordinates of any point on the circle give the values of the adjacent and opposite sides of a right angled triangle, with the radius the hypotenuse.

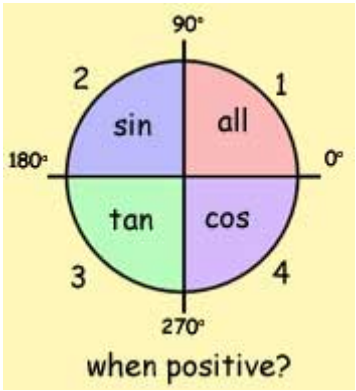
The General Angle (theta) is the included angle between the radius and the x-coordinate of the point.

As the radius rotates the x and y values change. Hence the values of sine, cosine and tangent also change.

$$\sin\theta = \frac{y}{1} \quad \cos\theta = \frac{x}{1} \quad \tan\theta = \frac{y}{x}$$



The result is summarized in the diagram below.



Example #1

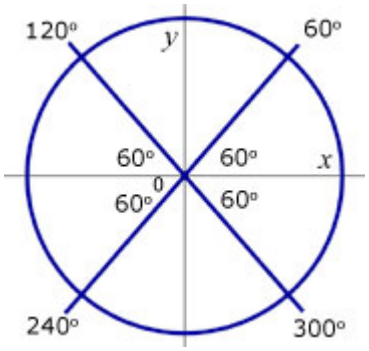
solve the equation $\sin\theta = -\frac{\sqrt{3}}{2}$ for $0 \le \theta \le 360$

$\sin^{-1}(\frac{\sqrt{3}}{2}) = 60°$

(meaning the angle whose sin is $\frac{\sqrt{3}}{2}$, is 60°)

the diagram shows the angles available with a a sine value of $\pm\frac{\sqrt{3}}{2}$

however, the sine value is only negative in the 3rd and 4th quadrants



therefore the roots of the equation for $0 \le \theta \le 360$ are: $\theta = 240°$ and $\theta = 300°$

Example #2

solve the equation $2\cos 2\theta = \sqrt{3}$ for $-180^\circ < \theta < 180^\circ$

rewriting $2\cos 2\theta = \sqrt{3}$,

$\cos 2\theta = \frac{\sqrt{3}}{2} = 0.8660$

$\cos^{-1} 2\theta = 60^\circ$

$\therefore \quad \theta = 30^\circ$

N.B. if $-180^\circ < \theta < 180^\circ$ then $-360^\circ < 2\theta < 360^\circ$

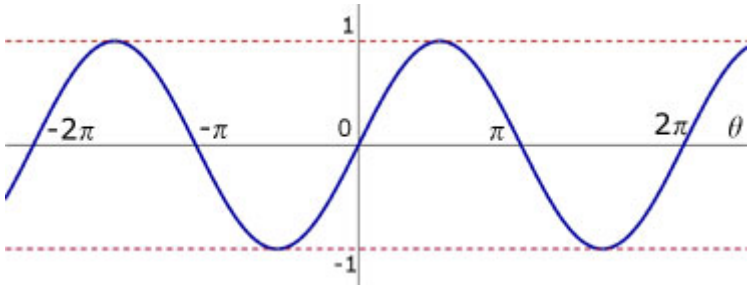
the cosine function is only + ve. in quadrants 1 & 4

$\therefore$ for $0 < 2\theta < 360^\circ \quad 2\theta = 60^\circ, 300^\circ, -60^\circ, -300^\circ$

$\Rightarrow$ for $0 < \theta < 180^\circ \quad \theta = 30^\circ, 150^\circ, -30^\circ, -150^\circ$

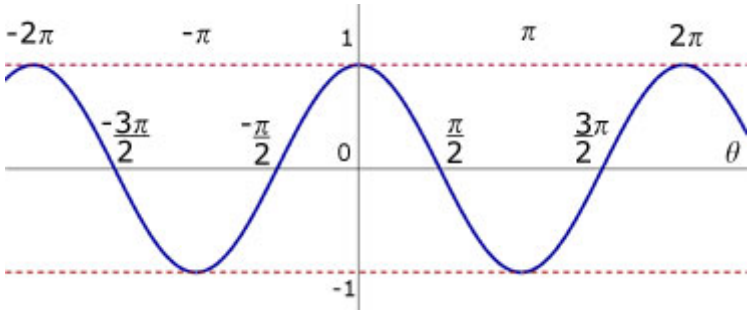
(remember + is an anticlockwise measure of angles
and - a clockwise measure)

Sine



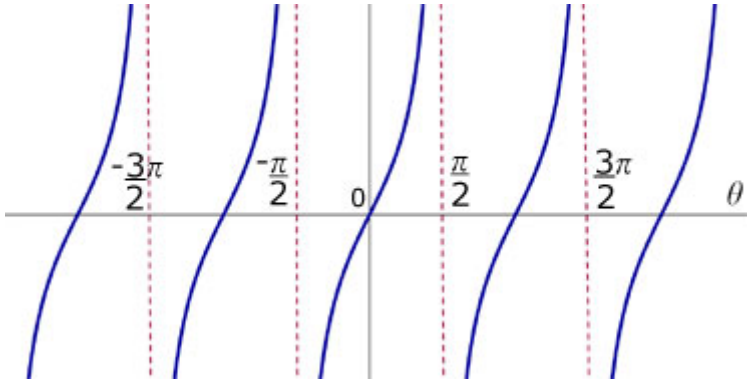
- the sine graph starts at **zero**
- it repeats itself every **360** degrees(or 2 pi)
- y is never more than **1** or less than **-1** (displacement from the x-axis is called the **amplitude**)
- a sin graph 'leads' a cos graph by 90 degrees

Cosine



- the cosine graph starts at **one**
- it repeats itself every **360** degrees(or 2 pi)
- y is never more than **1** or less than **-1** (displacement from the x-axis is called the **amplitude**)
- a cos graph 'lags' a sin graph by 90 degrees(pi/2) - this is termed a **phase shift**

Tangent



- the tangent graph starts at **zero**
- it repeats itself every **180** degrees
- y can vary between numbers approaching infinity and minus infinity

Further comparison

- only the cosine function is symmetrical about the y-axis
- all the functions are **cyclic** - the distance along the horizontal axis repeated is called **the period**