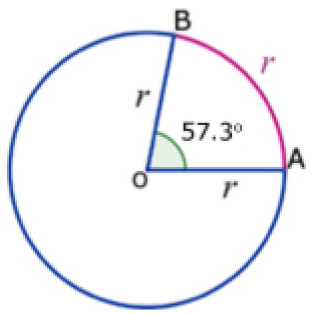
Radians

What is a 'radian'?

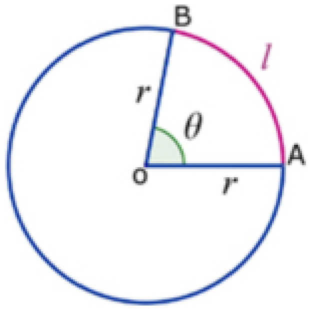


A radian is the angle subtended at the centre of a circle by an arc the same length as the radius of the circle.

Units

1^C (meaning 1 radian)= 57.296 deg.

Arc length



The arc length is proportional to its subtended angle.

Hence, if θ(theta) is in **degrees** and '*l*' is the arc length:

$$\frac{l}{\theta} = \frac{2\pi r}{360}$$

$$l = \frac{\theta}{360} \times 2\pi r$$

An angle can be expressed in **radians** by dividing the arc length by the radius.

Therefore θ in radians is given by:

$$\theta = \frac{l}{r}$$

$$\therefore \text{ arc length} \quad l = r\theta$$

Therefore for a circle(a 360 deg. angle), where the arc length is '2πr' and the radius is 'r' , the number of radians is 2πr/r , i.e. 2π .

Sector area

The area of a sector is proportional to the angle its arc subtends at the centre.

If a sector contains an angle of θ^o then its area is given by:

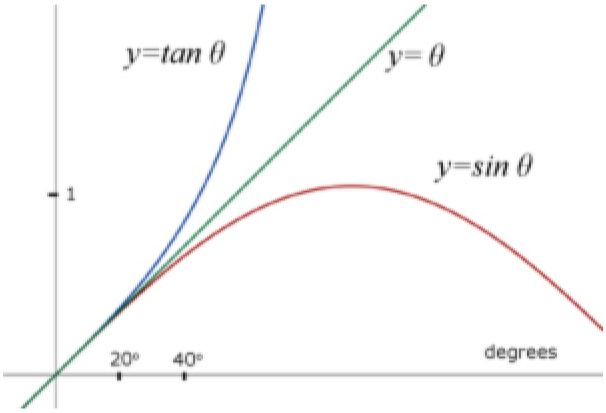
$$\text{area of sector } = \frac{\theta}{360} \times \pi r^2$$

However, if θ is in radians, remembering there are 2π radians in a circle:

$$\begin{aligned}\text{area of sector } &= \frac{\theta}{2\pi} \times \pi r^2 \\ &= \frac{\theta}{2} r^2\end{aligned}$$

$$\underline{\text{area of sector } = \frac{1}{2} r^2 \theta}$$

Small angles



For small angles(<10 deg.) there is a convergence between the value of the angle in radians with the value of its sine & tangent.

This approximate sine value may be expressed as:

$$\sin\theta \simeq \theta$$

The approximate cosine value is obtained thus:

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\therefore \quad \cos\theta = 1 - 2\sin^2(\tfrac{1}{2}\theta) \qquad \text{(i}$$

if θ is small, $\sin\tfrac{1}{2}\theta \simeq \tfrac{1}{2}\theta$

hence equation (i becomes

$$\cos\theta = 1 - 2(\tfrac{1}{2}\theta)^2$$

$$\therefore \quad \underline{\cos\theta = 1 - \tfrac{1}{2}\theta^2}$$