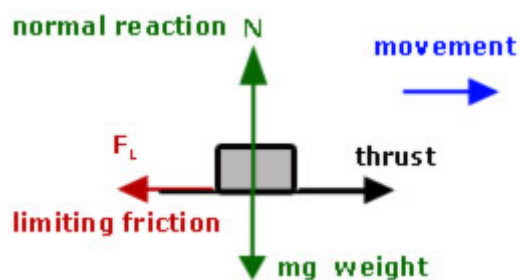


Statics : Friction

**Friction** is the force that opposes movement.

The 3 Laws of Friction

1.

The **limiting frictional force** (  $F_L$  ) is directly proportional to the **normal contact force** (  $N$  )

$$F_L \propto N$$

note: limiting frictional force is the maximum frictional force

2.

The ratio of the **limiting frictional force** (  $F_L$  ) to the **normal contact force** (  $N$  ) is called the coefficient of friction (  $\mu$  )

$$\mu = \frac{F_L}{N} \qquad F_L = \mu N$$

3.

When there is no motion, but the object is on the point of moving,

**applied force = frictional force**(limiting friction)

and when there is motion,

**applied force > frictional force**(limiting friction)

then this equality applies:

$$F_L = \mu N$$

Up to this point, when the frictional force is **less** than limiting friction(maximum)\*, then the inequality below applies.

$$F_L < \mu N$$

\*object is static and not on the point of moving

### Example #1

A flat stone is thrown horizontally across a frozen lake. If the stone decelerates at  $2.5 \text{ ms}^{-2}$ , what is the coefficient of friction between the stone and the ice? (take  $g=10 \text{ ms}^{-2}$ )

since the stone is moving, the friction is limiting(max.)

$$\therefore F_L = \mu N \quad (1)$$

and vertically,  $N = mg$

the stone is decelerating,

using Newton's Second law,

$$F_L = ma$$

substituting for  $F_L$  and  $N$  into (1)

$$ma = \mu mg$$

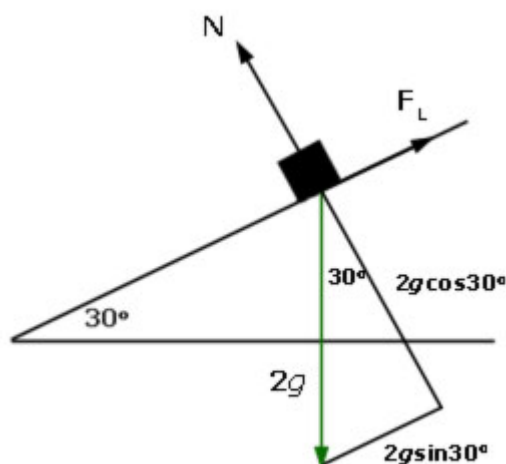
$$\Rightarrow \mu = \frac{ma}{mg} = \frac{a}{g} = \frac{2.5}{10} = 0.25$$

Ans. coefficient of friction between stone & ice is 0.25

Example #2

A 2 kg mass in limiting equilibrium rests on a rough plane inclined at an angle of  $30^\circ$  to the horizontal.

Show that the coefficient of friction between the mass and the plane is  $\frac{\sqrt{3}}{3}$ .



resolving along the plane

$$F_L = 2g \sin 30^\circ = 2g \cdot \frac{1}{2}$$

$$F_L = g$$

resolving at right angles to the plane

$$N = 2g \cos 30^\circ$$

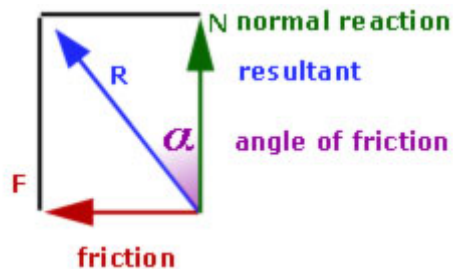
$$N = 2g \left( \frac{\sqrt{3}}{2} \right) = g\sqrt{3}$$

for limiting friction  $F_L = \mu N$

$$\Rightarrow \mu = \frac{F_L}{N} = \frac{g}{g\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Ans. coefft. of friction between 2 kg mass & plane is  $\frac{\sqrt{3}}{3}$

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The Angle of Friction

If we examine the normal reaction force(  $N$  ) and the frictional force(  $F_L$  ) when it is limiting, then the equation  $F_L = \mu N$  applies.

If the resultant between  $N$  and  $F_L$  is  $R$ , and it is inclined at an angle  $\alpha$ (alpha) to the normal  $N$ , then we can write equations for  $F_L$  and  $N$  in terms of  $R$ .

$$\frac{F_L}{N} = \mu$$

$$F_L = R \sin \alpha$$

$$N = R \cos \alpha$$

$$\frac{F_L}{N} = \frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha$$

$$\Rightarrow \underline{\mu = \tan \alpha}$$

In example #2(above) the angle of friction =  $\tan^{-1}(\sqrt{3} / 3)$   
 =  $\tan^{-1}(0.5773) = 30^\circ$  (the angle of the plane)