Statics : Particle Forces in Equilibrium

Below is a list of methods for describing forces in equilibrium acting on a particle. The forces act in **one** plane and are called **coplanar** . In this context the word **equilibrium** means that the forces are in balance and there is no net force acting.

Vector Method

The condition for equilibrium is that the vector sum of forces(the resultant)is the **null vector**. This means that the multipliers for the **i**, **j** and **k** unit vectors are each zero.

Example #1

A particle at rest has forces (2**i** + 3**j**), (m**i** + 6**j**) and (-4**i** + n**j**) acting on it.
What are the values of m and n ?

For equilibrium, all the forces when added together(the vector sum) equal zero.

$$2\boldsymbol{i} + 3\boldsymbol{j} + m\boldsymbol{i} + 6\boldsymbol{j} - 4\boldsymbol{i} + n\boldsymbol{j} = 0$$
$$(2 + m - 4)\boldsymbol{i} + (3 + 6 + n)\boldsymbol{j} = 0$$

The scalar multipliers of each unit vector equal zero.

$$2 + m - 4 = 0$$
$$m = 4 - 2$$
$$\underline{m = 2}$$

$$3 + 6 + n = 0$$
$$n = -6 - 3$$
$$\underline{n = -9}$$

Ans. m = 2 and n = -9

Example #2

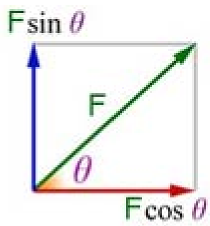
A particle at rest has an unknown force F and two other forces (2**i** - 5 **j**), (-3**i** + **j**), acting on it.
What is the magnitude of F ? (2 d.p.)
(all forces in Newtons, N)

for equilibrium the vector sum equals zero

$$2\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{F} = 0$$
$$\boldsymbol{F} = (2-3)\boldsymbol{i} + (-5+1)\boldsymbol{j}$$
$$\boldsymbol{F} = -\boldsymbol{i} - 4\boldsymbol{j}$$
$$|\boldsymbol{F}| = \sqrt{(-1)^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17} = 4.12$$

Ans. force F = 4.12 N

Resolution Method

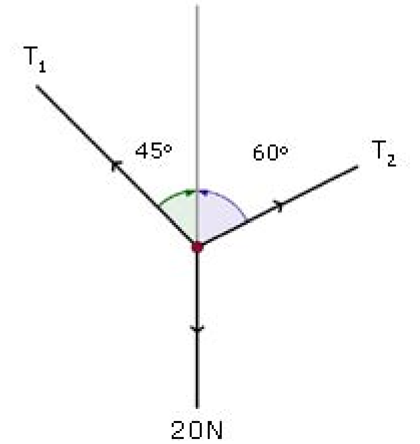


A force **F** can be replaced by two vectors that are at right angles to eachother, passing through the point of application. Hence if the angle between one component vector and the original vector is θ(theta), then the two components are **F**sinθ and **F**cosθ .

Problems are solved by resolving all the vectors into their horizontal and vertical components. The components are then resolved vertically and then horizontally to obtain two equation. These can be solved as simultaneous equations.

Example

A 2 kg mass is suspended by two light inextensible strings inclined at 60 deg. and 45 deg. to the vertical.
What are the tensions in the strings?(to 2 d.p.)
(assume g=10 ms^{-2})



resolving horizontally $T_1 \sin 60^\circ = T_2 \sin 45^\circ$

$$T_1 = T_2 \frac{\sin 45^\circ}{\sin 60^\circ} = \frac{0.7071}{0.8660} T_2 = 0.8165 T_2$$

$$T_1 = 0.8165 T_2 \qquad (i$$

resolving vertically

$$T_1 \cos 60^\circ + T_2 \cos 45^\circ = 20 \qquad (ii$$

substituting for T_1 in (ii

$$0.8165 T_2 (0.5) + 0.7071 T_2 = 20$$

$$0.4083 T + 0.7071 T_2 = 20$$

$$1.1154 T_2 = 20$$

$$T_2 = \frac{20}{1.1154} = 17.9308$$

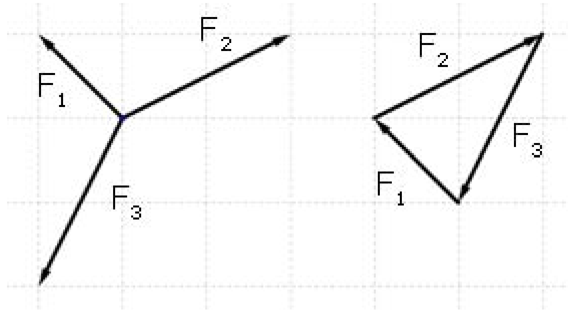
from (i $T_1 = 0.8165 T_2$

$$T_1 = 0.8165 \times 17.9308 = 14.6405$$

Ans. the tensions in the strings are 14.6 N and 17.9 N

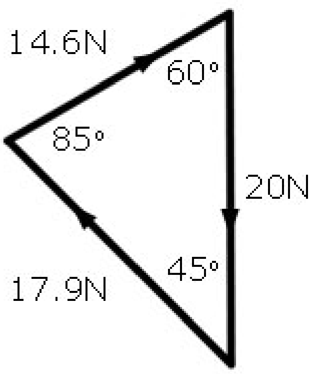
Triangle of Forces

When 3 coplanar forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the adjacent sides of a triangle taken in order.



Example

Using the results from the previous example, the three forces acting on the 2 kg mass can be represented by a scale diagram.



Our starting point is the 20N force acting downwards. One force acts at 45 deg. to this line and the other at 60 deg. So to find the magnitude of the two forces, draw lines at these angles at each end of the 20N force. Where the lines cross gives a vertex of the triangle. Measuring the lengths of the lines from this to the ends of the 20N force line will give the magnitudes of the required forces.

Polygon of Forces

For equilibrium, forces are represented in magnitude and direction to form a polygon shape.

If a number of forces are acting at a point, then the missing side in the polygon represents the resultant force. Note the arrow direction on this force is in the opposite direction to the rest.