Momentum & Impulse : Impulse

Momentum

Momentum is by definition the product of mass and velocity. So strictly speaking momentum is a vector quantity.

$$\text{momentum} = \text{mass(kg)} \times \text{velocity}(\text{ms}^{-1})$$

Hence the unit of momentum is (kg.ms^{-1}).

Impulse of a force

This is simply the force multiplied by the time the force acts.

We can obtain an expression for this in terms of momentum from Newton's Second Law equation $F=ma$, where the force F is constant.

Remembering that velocity, force and therefore impulse are vector quantities.

For a mass m being accelerated by a constant force $\mathbf{F}$, where the impulse is $\mathbf{J}$, $\mathbf{v}_1$ is initial velocity and $\mathbf{v}_2$ is final velocity:

$$\mathbf{F}t = m(\mathbf{v}_2 - \mathbf{v}_1)$$

$$\mathbf{J} = \mathbf{F}t$$

$$\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$$

Units

Since impulse is the product of force and time, the units of impulse are (Newtons) x (seconds), or N s .

Vector problems

Vector type questions on impulse are solved by first calculating the change in momentum. This gives a vector expression for the impulse. Using Pythagoras, the magnitude of the impulse can then be found. The anglular direction is calculated from the coefficients of unit vectors **i** and **j**.

Example #1

A particle of mass 0.5 kg moves with a constant velocity of (3**i** + 5**j**) $m.s^{-1}$.
After being given an impulse, the particle then moves off with a constant velocity of (2**i** - 3**j**) $m.s^{-1}$.

Calculate:

i) the impulse
ii) the magnitude of the impulse(to 2 d.p.)
iii) the direction of the impulse(θ degrees to the x-axis)

i)

$\mathbf{v}_1 = (3\mathbf{i} + 5\mathbf{j})$ $\quad \mathbf{v}_2 = (2\mathbf{i} - 3\mathbf{j})$ $\quad m = 0.5$ kg

using $\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$

$\mathbf{J} = 0.5(2\mathbf{i} - 3\mathbf{j}) - 0.5(3\mathbf{i} + 5\mathbf{j})$

$\mathbf{J} = \mathbf{i} - 1.5\mathbf{j} - 1.5\mathbf{i} - 2.5\mathbf{j}$

$\mathbf{J} = (1 - 1.5)\mathbf{i} + (-1.5 - 2.5)\mathbf{j}$

$\underline{\mathbf{J} = (0.5\mathbf{i} - 4\mathbf{j})\text{ N.s}}$

ii)

magnitude of impulse = $\sqrt{[(0.5)^2 + (-4)^2]} = \sqrt{[16.23]}$

= $\underline{4.03 \text{ N.s}}$

iii) direction $\tan^{-1}\theta = (4)/(0.5) = 8$

$\theta = 82.8749^{\circ} = \underline{82.87}^{\circ}$ (2 d.p.) clockwise to the x-axis

Example #2

A particle of mass 2.5 kg is moving with a constant velocity of $(2\mathbf{i} + \mathbf{j})\ \text{m.s}^{-1}$.
After an impulse, the particle moves with a constant velocity of $(4\mathbf{i} + 3\mathbf{j})\ \text{m.s}^{-1}$.

Calculate:

i) the impulse
ii) the magnitude of the impulse(to 2 d.p.)
iii) the angle(θ^{o}) the impulse makes with the x-axis

i)

$\mathbf{v}_1 = (2\mathbf{i} + \mathbf{j}) \quad \mathbf{v}_2 = (4\mathbf{i} + 3\mathbf{j}) \quad m = 2.5\ \text{kg}$

using $\mathbf{J} = m(\mathbf{v}_2 - \mathbf{v}_1)$

$\mathbf{J} = 2.5(4\mathbf{i} + 3\mathbf{j}) - 2.5(2\mathbf{i} + \mathbf{j})$

$\mathbf{J} = 10\mathbf{i} + 7.5\mathbf{j} - 5\mathbf{i} - 2.5\mathbf{j}$

$\mathbf{J} = (10 - 5)\mathbf{i} + (7.5 - 2.5)\mathbf{j}$

$\underline{\mathbf{J} = (5\mathbf{i} + 5\mathbf{j})}$

ii)

magnitude of impulse = $\sqrt{[(5)^2 + (5)^2]} = \sqrt{[50]} = \underline{7.07\ \text{N.s}}$

iii) direction $\tan^{-1}\theta = (5)/(5) = 1$

$\theta = \underline{45^{o}}$ anticlockwise to the x-axis