# Linear Motion : Uniform Acceleration

## Introduction

To understand this section you must remember the letters representing the variables:

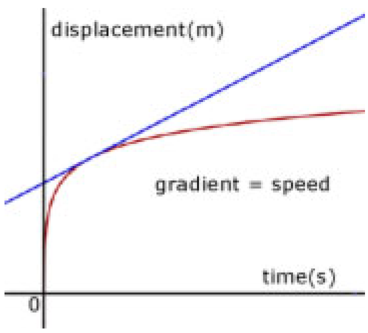
**u** - initial speed
**v** - final speed
**a** - acceleration(+) or deceleration(-)
**t** - time taken for the change
**s** - displacement(distance moved)

It is also important to know the **S.I. units** ( *Le Système International d'Unités*) for these quantities:

**u** - metres per second ($ms^{-1}$)
**v** - metres per second ($ms^{-1}$)
**a** - metres per second per second ($ms^{-2}$)
**t** - seconds (s)
**s** - metres (m)

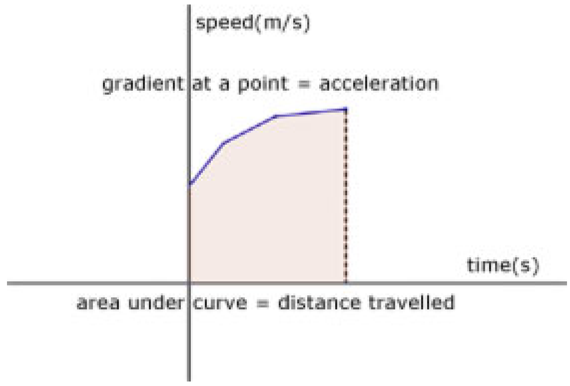
in some text books 'speed' is replaced with 'velocity'. Velocity is more appropriate when direction is important.

## Displacement-time graphs




For a displacement-time graph, the gradient at a point is equal to the **speed** .

## Speed-time graphs




For a speed-time graph, the area under the curve is the distance travelled.

The gradient at any point on the curve equals the acceleration.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Note, the acceleration is also the second derivative of a speed-time function.

## Equations of Motion

One of the equations of motion stems from the definition of acceleration:

acceleration = the rate of change of speed

$$a = \frac{v - u}{t}$$

rearranging

$$v = u + at \qquad (1$$

if we define the distance 's' as the average speed times the time(t), then:

$$s = \left(\frac{u+v}{2}\right)t$$

rearranging

$$u + v = \frac{2s}{t}$$

rearranging (i

$$v - u = at$$

subtracting these two equations to eliminate *v*

$$2u = \frac{2s}{t} - at$$

$$2ut = 2s - at^2$$

$$2ut + at^2 = 2s$$

$$ut + \frac{at^2}{2} = s, \qquad s = ut + \frac{at^2}{2}$$

it is left to the reader to show that :

$$v^2 - u^2 = 2as$$

hint: try multiplying the two equations instead of subtracting

summary:

$$s = \left(\frac{u+v}{2}\right)t$$

$$v - u = at$$

$$s = ut + \frac{at^2}{2}$$

$$v^2 - u^2 = 2as$$

Example #1

A car starts from rest and accelerates at 10 $ms^{-1}$ for 3 secs.
What is the maximum speed it attains?

$$u = 0 \text{ ms}^{-1} \qquad a = 10 \text{ ms}^{-2} \qquad t = 3 \text{ s}$$

$$\begin{aligned} v &= u + at \\ &= 0 + (10 \times 3) \\ &= 30 \end{aligned}$$

Ans. maximum speed is 30 $ms^{-1}$

Example #2

A car travelling at 25 $ms^{-1}$ starts to decelerate at 5 $ms^{-2}$.
How long will it take for the car to come to rest?

$$u = 25 \text{ ms}^{-1} \qquad v = 0 \qquad a = -5 \text{ ms}^{-2}$$

$$\begin{aligned} v &= u + at \\ 0 &= 25 + (-5)t \\ 5t &= 25 \\ t &= 5 \end{aligned}$$

Ans. it takes 5 secs. for the car to stop

Example #3

A car travelling at 20 $ms^{-1}$ decelerates at 5 $ms^{-2}$.
How far will the car travel before stopping?

$$u = 20 \text{ ms}^{-1} \qquad v = 0 \qquad a = -5 \text{ ms}^{-2}$$

$$\begin{aligned} v^2 - u^2 &= 2as \\ 0 - (20 \times 20) &= 2(-5)s \\ -400 &= -10s \\ s = \frac{400}{10} &= 40 \end{aligned}$$

Ans. distance travelled is 40 m

Example #4

A car travelling at 30 $ms^{-1}$ accelerates at 5 $ms^{-2}$ for 8 secs.
How far did the car travel during the period of acceleration?

$$u = 30\ \text{ms}^{-1} \qquad a = 5\ \text{ms}^{-2} \qquad t = 8\ \text{s}$$

$$s = ut + \tfrac{1}{2}at^2$$

$$\begin{aligned} s &= (30\times 8) + \tfrac{1}{2}(5\times 8\times 8) \\ &= 240 + 160 \\ &= 400 \end{aligned}$$

Ans. car travelled 400 m during acceleration

Vertical motion under gravity

These problems concern a particle projected vertically upwards and falling 'under gravity'.

In these types of problem it is assumed that:

air resistance is minimal

displacement & velocity are positive(+) upwards & negative(-)downwards

acceleration(g) always acts downwards and is therefore negative(-)

acceleration due to gravity(g) is a constant

Example #1

A stone is thrown vertically upwards at 15 $ms^{-1}$.

(i) what is the maximum height attained?
(ii) how long is the stone in the air before hitting the ground?

(Assume g = 9.8 $ms^{-2}$. Both answers to 2 d.p.)

$u = 15\ \text{ms}^{-1} \qquad a = -9.8\ \text{ms}^{-2}$

max. height is when final velocity $v = 0$

$$v^2 - u^2 = 2as$$
$$0 - (15)^2 = 2(-9.8)s$$
$$-225 = -19.6s$$
$$s = \frac{225}{19.6} = 11.4796$$

Ans. max. height is 11.48 m (2 d.p.)

$u = 15\ \text{ms}^{-1} \qquad a = -9.8\ \text{ms}^{-2}$

time of flight is $2t$, twice time to max. height $t$
max. height is when final velocity $v = 0$

$$v = u + at$$
$$= 15 + (-9.8)t$$
$$15 = 9.8t$$
$$t = \frac{15}{9.8} = 1.5306$$
$$\therefore \quad 2t = 3.0612$$

Ans. max. time in air is 3.06 secs. (2 d.p.)

Example #2

A boy throws a stone vertically down a well at 12 $ms^{-1}$.
If he hears the stone hit the water 3 secs. later,

(i) how deep is the well?
(ii)what is the speed of the stone when it hits the water?

(Assume g = 9.8 $ms^{-2}$. Both answers to 1 d.p.)

$$u = -12 \text{ ms}^{-1} \qquad a = -9.8 \text{ ms}^{-2} \quad t = 3 \text{ s}$$

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ &= (-12)(3) + \tfrac{1}{2}(-9.8)(3 \times 3) \\ &= -36 - 44.1 \\ &= -80.1 \end{aligned}$$

Ans. depth of well is 80.1 m (1 d.p.)

$$\begin{aligned} v &= u + at \\ &= (-12) + (-9.8)(3) \\ &= -121 - 29.4 \\ &= -41.4 \end{aligned}$$

Ans. stone strikes water at 41.4 $\text{ms}^{-1}$(1 d.p.)