Linear Motion : Non-uniform Acceleration

Theory

Consider a particle P moving in a straight line from a starting point O.

The displacement from O is x at time t .
The initial conditions are: $t \geq 0$ when $x=0$.
if v is the velocity of P at time t, then

$$v = \frac{dx}{dt}$$

The acceleration 'a' of particle P is defined as:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

or alternately,

$$a = \frac{dv}{dt}$$
$$= \frac{dv}{dx}\frac{dx}{dt}$$

but $\frac{dx}{dt} = v$

$$\therefore \quad a = v\frac{dv}{dx}$$

Problems on this topic are solved by analysing the information given to form a differential equation. This is then integrated, usually between limits.

Example #1

A particle moves in a straight line such that its acceleration 'a' at time 't' is given by:

$$a = 4t - 7$$

If the initial speed of the particle is 5 ms^{-1}, at what values of 't' is the particle stationary?

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = 4t - 7$$

$$\Rightarrow \quad dv = (4t - 7)dt$$

integrating both sides

$$\int dv = \int (4t - 7)dt$$

$$\Rightarrow \quad v = 2t^2 - 7t + C$$

but $v = 5\ ms^{-1}$ when $t = 0$

$$\Rightarrow \quad 5 = 0 - 0 + C$$

$$\therefore \quad C = 5$$

$$\Rightarrow \quad v = 2t^2 - 7t + 5$$

$$= (2t - 5)(t - 1)$$

the particle is at rest when $v = 0$

$$\therefore \quad (2t - 5)(t - 1) = 0$$

$$\Rightarrow \quad t = 5/2, \quad t = 1$$

particle is stationary at $t = 1$ sec. $t = 2.5$ sec.

Example #2

A particle moves from a point O in a straight line with initial velocity 4 ms^{-1}.
if v is the velocity at any instant, the acceleration a of the particle is given by:

$$a = \frac{3}{v}$$

The particle passes through a point X with velocity 8 ms^{-1}.

(i) how long does the particle take to reach point X?
(ii) what is the distance OX?(1 d.p.)

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{3}{v}$$

$$\Rightarrow \quad dt = \frac{v}{3}dv$$

integrating both sides

$$\int dt = \int \frac{v}{3}dv$$

the limits of v are 8 ms^{-1} and 4 ms^{-1}

$$t = \left[\frac{v^2}{6}\right]_4^8$$

$$= \left[\frac{64}{6}\right] - \left[\frac{16}{6}\right]$$

$$= \frac{48}{6} = 8$$

Ans. particle takes 8 secs. to reach X

$$v\frac{dv}{dx} = \frac{3}{v}$$

$$\Rightarrow \qquad dx = \frac{v^2}{3}dv$$

integrating both sides

$$\int dx = \int \frac{v^2}{3}dv$$

the limits of v are $8ms^{-1}$ and $4ms^{-1}$

$$x = \left[\frac{v^3}{9}\right]_4^8$$

$$= \left[\frac{512}{9}\right] - \left[\frac{64}{9}\right]$$

$$= \frac{448}{9} = 49.\dot{7}$$

Ans. distance OX is 49.8 metres (1 d.p.)