Kinetics : Work & Energy

Theory - Work & Energy

Consider a particle of mass m moving linearly with an applied force F constantly acting on it.
u = initial speed, v = final speed, a = acceleration,
s = distance covered, t = time taken

from Newton's 2nd Law,

$$F = ma \qquad (1$$

from equations describing uniform linear motion

$$v^2 - u^2 = 2as$$

$$\Rightarrow \qquad a = \frac{v^2 - u^2}{2s}$$

substituting for a in equation (1

$$F = m\left(\frac{v^2 - u^2}{2s}\right)$$

$$= \frac{1}{s}\left[\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2\right]$$

$$\underline{Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2}$$

By definition,

work done = (force) x (distance force moves)

Since the expression ½mv^2 is defined as the kinetic energy of a particle of mass m, speed v, our definition is modified to:

work done = change in kinetic energy produced

mathematical proof

Consider a particle, mass m, speed v, being moved along the x-axis by a force of magnitude F.
The applied force F is proportional to the displacement of the particle, x, along the x-axis.

$$F = ma$$

since $a = v\dfrac{dv}{dx}$

$$\Rightarrow \quad F = mv\frac{dv}{dx}$$

$$\Rightarrow \quad Fdx = mvdv$$

$$\Rightarrow \quad \int Fdx = \int mvdv$$

considering the work done when the particle has:
original velocity v_1 at displacement x_1
final velocity v_2 at displacement x_2

$$\Rightarrow \quad \int_{x_1}^{x_2} Fdx = \int_{v_1}^{v_2} mvdv$$

$$F\left[x\right]_{x_1}^{x_2} = \left[\tfrac{1}{2}mv^2\right]_{v_1}^{v_2}$$

$$\underline{F(x_2 - x_1) = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}mv_1^2}$$

Gravitational Potential Energy

This is the energy a mass posesses by virtue of its position. It is equal to the product of mass, gravitational field strength(g) and the vertical distance the particle is above a fixed level.

m = mass speed(ms^{-1}), g = gravitational field strength(N/kg)

h = vertical distance(m)

$$\text{potential energy P.E.(joules)} = mgh$$

(note the unit for g - the gravitational force on a mass of 1kg)

The Law of Conservation of Energy

In a closed system the amount of energy is constant. Or in other words 'energy can never be created nor destroyed', it mearly changes from one form into another.

This is the classical physics view that is useful for most purposes. However, in the real world systems are seldom perfect. We also have the problem when referring to particle physics that energy can indeed be created and destroyed. Annihilation of elementary particles is an example of this(matter-antimatter: electron-positron collision).

Example #1

A pump forces up water at a speed of $8ms^{-1}$ from a well into a reservoir at a rate of 50 kg s^{-1} .
If the water is raised a vertical height of 40 m, what is the work done per second?(assume $g=10\ ms^{-2}$)

$$m = 50\text{kg} \quad h = 40\text{m} \quad g = 10\text{ms}^{-2} \quad v = 8\text{ms}^{-1}$$

$$\begin{aligned}\text{work done} &= \text{increase in PE} + \text{increase in KE}\\ &= mgh + \tfrac{1}{2}mv^2\\ &= (50\times10\times40)+(0.5\times50\times8\times8)\\ &= 20,000+1,600 = 21,600\end{aligned}$$

Ans. work done per second is 21,600J.

Example #2

A gun is fireed at a 3 cm thick solid wooden door.
The bullet, of mass 7g, travels through the door and has speed reduced from 450 ms^{-1} to 175 ms^{-1}.
Assuming uniform resistance, what is the force of the wood on the bullet. (answer to 5 sig. figs.)

$u = 450\text{ms}^{-1} \quad v = 175\text{ms}^{-1} \quad s = 3\text{cm} \equiv 0.03\text{m}$

$m = 7\text{g} \equiv 0.007\text{kg}$

using $v^2 - u^2 = 2as$

$$a = \frac{v^2 - u^2}{2s}$$

$$= \frac{(175)^2 - (450)^2}{2 \times 0.03}$$

$$= \frac{30625 - 202500}{0.06}$$

$$= 3885416.667$$

using $F = ma,$

$$F = 0.007 \times 3885416.667$$

$$= 27197.917$$

Ans. resistive force from wood is 27198N (5 sig.figs.)

Example #3

In a science experiment, a 50g mass slides down a 60^o incline of length 0.5m.
If the mass is given an initial speed of 2 ms^{-1} down the plane and its final speed is measured as 3 ms^{-1}, what is the magnitude of the frictional force opposing the mass?
(assume g=10 ms^{-2} , answer to 2d.p.)

$m = 50\text{g} \equiv 0.05\text{kg} \quad s = 0.5\text{m} \quad u = 2\text{ms}^{-1} \quad v = 3\text{ms}^{-1}$

f frictional force

$$\text{energy at start} = \text{energy at end}$$

$$PE_{\textbf{start}} + KE_{\textbf{start}} = PE_{\textbf{end}} + KE_{\textbf{end}} + \text{work done}$$

$$mgh + \tfrac{1}{2}mu^2 = 0 + \tfrac{1}{2}mv^2 + fS$$

$$(0.05 \times 10 \times 0.433) + (0.5 \times 0.05 \times 4) = (0.5 \times 0.05 \times 9) + 0.5f$$

$$0.2165 + 0.1 = 0.225 + 0.5f$$

$$0.3165 - 0.225 = 0.5f$$

$$f = \frac{0.915}{0.5} = 1.83$$

Ans. frictional force opposing mass movement is 1.83N