

Kinetics : Elastic StringsHooke's Law

The extension  $e$  of an elastic string/spring is proportional to the extending force  $F$ .

$$F \propto e$$

$$F = ke$$

where  $k$  is the constant of proportionality, called the spring constant.

By definition the **spring constant** is that force which will produce unit extension (unit  $\text{Nm}^{-1}$ ) in a spring.

example

A 2kg mass hangs at the end of an elastic string of original length 0.5 m.  
If the spring constant is  $60 \text{ Nm}^{-1}$ , what is the length of the extended elastic string?

$$m = 2\text{kg} \quad k = 60\text{Nkg}^{-1} \quad l_o = 0.5\text{m}$$

$$e = l - l_o \quad g = 10\text{ms}^{-2}$$

resolving forces vertically on mass

$$mg = ke$$

substituting for  $e = l - l_o$

$$mg = k(l - l_o)$$

$$\frac{mg}{k} = (l - l_o)$$

$$l = l_o + \frac{mg}{k}$$

$$l = 0.5 + \left( \frac{2 \times 10}{60} \right) = 0.8\dot{3}$$

Ans. length of extended string is 0.8 m(1 d.p.)

The energy stored in a stretched string/spring  
(same as the work done in stretching a string)

Remembering that 'work = force x distance moved', if the elastic string is extended a small amount  $\delta e$  (such that the force  $F$  is considered constant) then the work done  $\delta W$  is given by:

$$\delta W = F \delta e \quad (1)$$

substituting for  $F = ke$  in (1)

$$\Rightarrow \delta W = ke \delta e$$

work  $W$  done in extension 0 to  $e$

$$\begin{aligned} W &= \int_0^e kede \\ &= k \int_0^e ede \\ &= k \left[ \frac{e^2}{2} \right]_0^e \\ &= \frac{ke^2}{2} \end{aligned}$$

$$\underline{W = \frac{1}{2}ke^2}$$

### example

A 2 kg mass  $m$  hangs at a point B at the end of an elastic string of natural length 0.7 m supported at a point A.

The extension produced in the string by the mass is a fifth of the original length, while the spring constant is  $5mg$ .

If the mass is now held at point A and then released, what is the maximum speed the mass will attain?

( $g = 10 \text{ ms}^{-2}$ , answer to 1 d.p.)

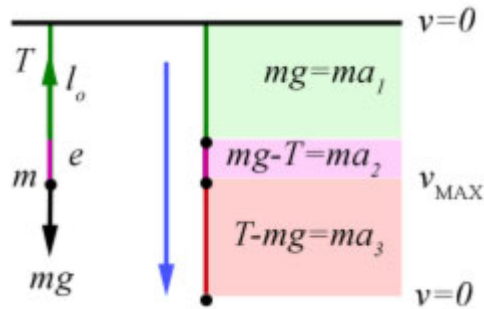
Let PE be related to the equilibrium position of the mass. Hence the mass will have maximum KE when passing through point B. Below that point there will be no net force downwards to produce acceleration. The net force will act in the opposite direction, producing deceleration.

In the diagram,

$a_1$  is linear acceleration under gravity

$a_2$  is non-linear acceleration(decreasing)

$a_3$  is negative acceleration i.e. deceleration.



$$m = 2\text{kg} \quad l_0 = 0.7\text{m} \quad e = \frac{l_0}{5} \quad l = \frac{6}{5} l_0$$

$$g = 10\text{ms}^{-2} \quad k = 5mg$$

$$\text{PE at A} = (\text{KE at B}) + (\text{elastic PE at B})$$

$$mgl = \frac{1}{2}mv^2 + \frac{1}{2}ke^2$$

$$\frac{6}{5}mgl_0 = \frac{1}{2}mv^2 + \frac{5}{2}mg\left(\frac{l_0}{5}\right)^2$$

$$\frac{6}{5}gl_0 = \frac{v^2}{2} + \frac{gl_0^2}{10}$$

multiplying both sides by 10

$$12gl_0 = 5v^2 + gl_0^2$$

$$5v^2 = 12gl_0 + gl_0^2$$

$$v = \sqrt{\frac{12gl_0 + gl_0^2}{5}}$$

$$= \sqrt{\frac{(12 \times 10 \times 0.7) + (10 \times 0.7 \times 0.7)}{5}}$$

$$= 4.2166$$

Ans. maximum speed of mass is  $4.2\text{ms}^{-1}$  (1d.p.)