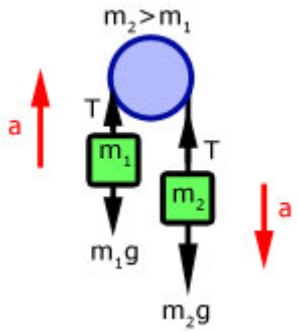
Kinetics : Connected Particles

Pulleys

Problems involve two weights either side of a pulley. The heavier weight pulls on the lighter causing both to accelerate in one direction with a common acceleration.



calculation of acceleration 'a'

$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$\underline{m_2 g - T = m_2 a}$$

$$m_2 g - m_1 g = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_2)}{(m_1 + m_2)} g$$

calculation of tension 'T'

$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$a = \frac{T - m_1 g}{m_1} \quad \text{(i}$$

$$m_2 g - T = m_2 a$$

$$a = \frac{m_2 g - T}{m_2} \quad \text{(ii}$$

equating (i and (ii

$$\frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$$

$$m_2 T - m_1 m_2 g = m_1 m_2 g - m_1 T$$

$$m_2 T + m_1 T = m_1 m_2 g + m_1 m_2 g$$

$$T(m_2 + m_1) = 2 m_1 m_2 g$$

$$T = \frac{2 m_1 m_2 g}{(m_2 + m_1)}$$

Example

A 3 kg mass and a 5 kg mass are connected over a pulley by a light inextensible string. When the masses are released from rest, what is:
i) the acceleration of each mass?
ii) the tension in the string
(Take g=9.8 ms^{-2} . Answer to 2 d.p.)

i)

$m_1 = 3\text{ kg} \quad m_2 = 5\text{ kg} \quad g = 9.8\text{ ms}^{-2}$

for mass m_1, using $f = ma$

$$T - m_1 g = m_1 a$$

$$T - 3g = 3a \qquad \text{(i}$$

for mass m_2, using $f = ma$

$$m_2 g - T = m_2 a$$

$$5g - T = 5a \qquad \text{(ii}$$

adding equations (i and (ii

$$T - 3g = 3a$$

$$\underline{5g - T = 5a}$$

$$2g = 8a$$

$$a = \frac{g}{4} = \frac{9.8}{4} = 2.45$$

Ans. acceleration of each mass is 2.45 ms^{-2}

ii)

from equation (i above

$$T - 3g = 3a$$

$$T = 3a + 3g$$

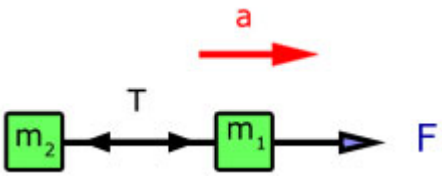
substituting for the values of g and a

$$T = (3 \times 2.45) + (3 \times 9.8) = 36.75$$

Ans. the tension in the string is 36.75 N

Towe-bar/towe-rope/chains

Usually one body pulled horizontally by another with each linked by a towe-bar or similar. This is similar to the pulley but drawn out in a line.



assuming no friction,

calculation of acceleration 'a'

$$T = m_2 a \qquad \text{(i}$$

$$F - T = m_1 a \qquad \text{(ii}$$

adding (i and (ii

$$F = (m_1 + m_2)a$$

$$a = \frac{F}{(m_1 + m_2)}$$

calculation of tension 'T'

from (i $T = m_2 a$

substituting for a

$$T = \frac{m_2 F}{(m_1 + m_2)}$$

Example

A car of mass 600 kg towes a trailer of mass 250 kg in a straight line using a rigid towe-bar.
The resistive force on the car is 200N.
The resistive force on the trailer is 80N.
If the forward thrust produced by the engine of the car is 800 N, what is(to 3 d.p.)
i) the acceleration of the car
ii) the tension in the towe-bar

i) looking at all the forces on the car and trailer together

thrust - resistance forces = total mass x acceleration
(tensions forward and backwards cancel out)

$$800 - 80 - 200 = (600 + 250)a$$
$$520 = 850a$$
$$a = \frac{520}{850} = 0.612$$

Ans. acceleration of car is 0.612 ms^{-2}

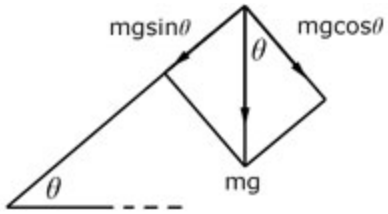
ii)

using $f = ma$ for the trailer only
$$T - 80 = 250a$$
$$T = 250a + 80$$
$$T = (250 \times 0.612) + 80 = 233$$

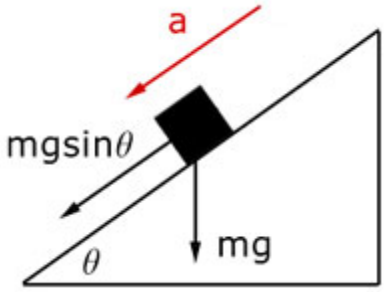
Ans. tension in towe-bar is 233N

Inclined plane with pulley

The pulley at the end just changes the direction of the force. problems involve the resolved component of the weight of the object down the plane.

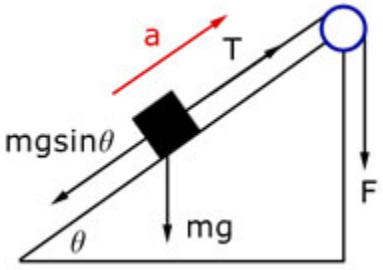


for a mass sliding down a smooth incline



$$mg\sin\theta = ma$$

for a mass pulled up an incline via a pulley



$$F - mg\sin\theta = ma$$

Example

A 2 kg mass on a smooth 30° plane is connected to a 5 kg mass by a light inextensible string passing over a pulley at the top of the plane.
When the particles are released from rest the 2 kg mass moves up the plane.
i) what is the acceleration of the 2 kg & 5 kg masses?
ii) What is the tension in the string?

i)

T tension in string
m_1 mass being pulled up incline
m_2 mass falling under gravity
a acceleration of both masses
$m_1 = 2$ kg $\quad m_2 = 5$ kg $\quad g = 9.8\ \text{ms}^{-2}$

consider forces on the 2kg mass, using $f = ma$

$$T - m_1 g \sin 30^\circ = m_1 a$$
$$T - (2 \times 9.8 \times 0.5) = 2a$$
$$T - 9.8 = 2a \qquad \text{(i}$$

consider forces on the kg mass, using $f = ma$

$$m_2 g - T = m_2 a$$
$$(5 \times 9.8) - T = 5a$$
$$49 - T = 5a \qquad \text{(ii}$$

adding equations (i and (ii

$$T - 9.8 = 2a$$
$$\underline{49 - T = 5a}$$
$$49 - 9.8 = 7a$$
$$a = \frac{39.2}{7} = 5.6$$

Ans. acceleration of the 2 kg & 5kg masses is 5.6 ms^{-2}

ii)

substituting into equation (i for a

$$T - 9.8 = 2a$$

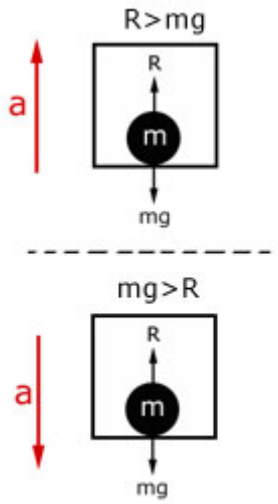
$$T = 2a + 9.8$$

$$= (2 \times 5.6) + 9.8 = 21$$

Ans. tension in string is 21N

Mass ascending or descending in a lift

It is important to remember that there are only two forces on the body in the lift - the weight down and the reaction of the floor up.



Example

A person of mass 100 kg stands in a lift.
What is the force exerted by the lift floor on the person when the lift is:
i) moving upwards at 3 ms^{-1}
ii) moving downwards at 4 ms^{-1}

i)

R reaction upwards of the lift floor

a acceleration of the lift upwards

m mass of person

$m = 100$ kg $\quad a = 3ms^{-1}$ $\quad g = 9.8\ ms^{-1}$

considering movement upwards, using $f = ma$

$$R - mg = ma$$

$$R = ma + mg$$

$$= (100 \times 3) + (100 \times 9.8)$$

$$= 300 + 980 = 1280$$

Ans. reaction of floor, lift ascending, is 1280 N

ii)

a acceleration of the lift downwards

$a = 4\ ms^{-1}$

considering movement downwards, using $f = ma$

$$mg - R = ma$$

$$R = mg - ma$$

$$= (100 \times 9.8) - (100 \times 4)$$

$$= 980 - 400 = 580$$

Ans. reaction of floor, lift descending, is 580 N