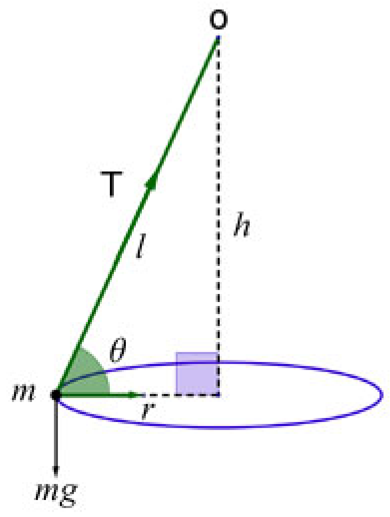
# Kinetics : Circular Motion

## Conical pendulum

Problems concerning the conical pendulum assume no air resistance and that the string has no mass and cannot be stretched.

Solution of problems involves resolving forces on the mass vertically and horizontally. In this way the speed of the mass, the tension in the string and the period of revolution can be ascertained.



resolving forces vertically on the mass

$$mg = T\sin\theta$$

$$T = \frac{mg}{\sin\theta} \qquad (i$$

resolving forces horizontally on the mass

$$T\cos\theta = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r\cos\theta} \qquad (ii$$

eliminating $T$ by combining (i and (ii

$$\frac{mg}{\sin\theta} = \frac{mv^2}{r\cos\theta}$$

$$gr = v^2\tan\theta$$

$$v = \sqrt{\frac{gr}{\tan\theta}}$$

but $\tan\theta = \frac{h}{r}$

$$\Rightarrow \qquad v = \sqrt{gr\left(\frac{r}{h}\right)}$$

$$\Rightarrow \qquad v = \sqrt{\left(\frac{gr^2}{h}\right)}$$

$$\Rightarrow \qquad \underline{v = r\sqrt{\left(\frac{g}{h}\right)}}$$

$$\text{the period of revolution} = \frac{\text{circumference}}{\text{speed}}$$
$$= \frac{2\pi r}{v}$$

substituting for $v$ using $v = r\sqrt{\frac{g}{h}}$

$$\text{the period of revolution} = \frac{2\pi r}{r\sqrt{\frac{g}{h}}} = \frac{2\pi r}{r}\sqrt{\frac{h}{g}}$$

$$\underline{\text{the period of revolution} = 2\pi\sqrt{\frac{h}{g}}}$$

Example

A 20g mass moves as a conical pendulum with string length $8x$ and speed $v$.
if the radius of the circular motion is $5x$ find:
i) the string tension(assume $g$ =10 ms$^{-2}$ , ans. to 2 d.p.)
ii) $v$ in terms of $x$, $g$

i)

$$l = 8x \qquad r = 5x \qquad \cos^{-1}\theta = \tfrac{5}{8} \qquad \theta = 51.3^\circ$$
$$m = 20\text{g} \equiv 0.02\text{kg} \qquad g = 10\text{ms}^{-2}$$

resolving vertically

$$T\sin\theta = mg$$
$$T = \frac{mg}{\sin\theta}$$
$$= \frac{0.02\times 10}{0.7804} = 0.2563$$

Ans. the string tension $T$ is 0.26N (2.d.p.)

ii)

resolving horizontally

$$T\cos\theta = \frac{mv^2}{r}$$

substituting for $T$, from $T = \frac{mg}{\sin\theta}$ above

$$mg\frac{\cos\theta}{\sin\theta} = \frac{mv^2}{r}$$

$$\frac{gr}{\tan\theta} = v^2$$

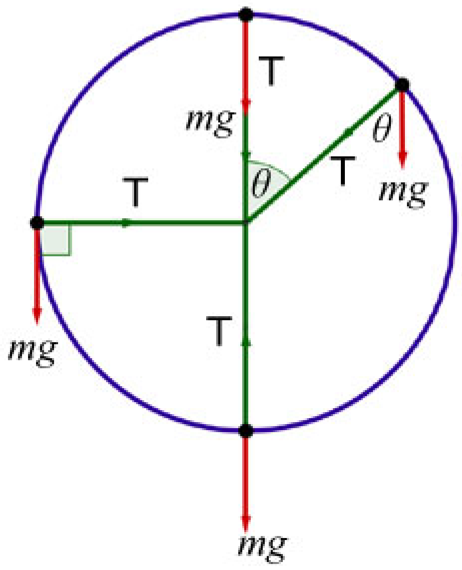
$$v = \sqrt{\frac{gr}{\tan\theta}}$$

substituting for $r = 5x$, $\theta = 51.3^\circ$

$$v = \sqrt{\frac{5gx}{\tan(51.3^\circ)}} = \sqrt{\frac{5}{\tan(51.3^\circ)}}.\sqrt{gx} = 2\sqrt{gx}$$

Ans. velocity $v$ in terms of $g$, $x$ is $2\sqrt{gx}$

Mass performing vertical circular motion under gravity




Consider a mass $m$ performing circular motion under gravity, the circle with radius $r$ .
The centripetal force on the mass varies at different positions on the circle.

top $mg + T = \frac{mv^2}{r}$

middle $T = \frac{mv^2}{r}$

bottom $T - mg = \frac{mv^2}{r}$

string at an angle $\theta$ to the vertical

$$mg\cos\theta + T = \frac{mv^2}{r}$$

For many problems concerning vertical circular motion, energy considerations(KE & PE) of particles at different positions are used to form a solution.

Example #1

A 50g mass suspended at the end of a light inextensible string performs vertical motion of radius 2m.
if the mass has a speed of 5 $ms^{-1}$ when the string makes an angle of $30^o$ with the vertical, what is the tension?
(assume $g$ =10 $ms^{-2}$ , answer to 1 d.p.)

$m = 50\text{g} \equiv 0.05\text{kg} \quad v = 5\text{ms}^{-1} \quad \theta = 30^\circ \quad r = 2\text{m}$

$g = 10\text{ms}^{-2}$

the centripetal force is the sum of the tension in the string and the component of the weight along the string

$$\Rightarrow \quad mg\cos\theta + T = \frac{mv^2}{r}$$

$$\Rightarrow \quad T = \frac{mv^2}{r} - mg\cos\theta$$

$$= \frac{(0.05)(5)^2}{2} - (0.05)(10)\cos 30^\circ$$

$$= 0.625 - 0.433 = 0.192$$

Ans. tension in string is 0.2N

Example #2

A 5kg mass performs circular motion at the end of a light inextensible string of length 3m. If the speed of the mass is 2 $ms^{-1}$ when the string is horizontal, what is its speed at the bottom of the circle?
(assume $g$ =10 $ms^{-2}$)

$v_H = 2ms^{-1}$ $\quad r = 3m$ $\quad g = 10ms^{-1}$

$v_B$ speed at bottom of circle

PE is measured relative to the bottom of the circle

KE + PE string horizontal = KE + PE at bottom

$$\tfrac{1}{2}mv_H^2 + mgr = \tfrac{1}{2}mv_B^2 + 0$$

$$v_H^2 + 2gr = v_B^2$$

$$v_B = \sqrt{v_H^2 + 2gr}$$

$$= \sqrt{(2)^2 + 2 \times (10) \times (3)}$$

$$= \sqrt{4 + 60} = \sqrt{64} = 8$$

Ans. speed at bottom of circle is 8 $ms^{-1}$