

Integration: The Integration Formula

The Integration Formula

The expression to be integrated is the derivative of some function eg $f(x)$ called the **integrand**.

When this expression is integrated the original function is restored plus a constant (C) called the constant of integration.

This is called the **indefinite integral** when the integration is not between two limiting values of x .

However when the integration is between two limiting values of x then the integral is called the **definite integral** and the constant of integration is not involved.

For any variable 'x' to the power of 'n' the integral is given by:

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

In other words, increase the power of x by '1' and divide x by the new index.

Rule #1

Any constant(eg C) multiplied by a function $f(x)$ can be integrated by placing the constant before the integration sign.

$$\int Cf(x)dx = C \int f(x)dx$$

Example

$$\int 5 \sin(x) dx = 5 \int \sin(x) dx = -5 \cos(x)$$

Rule #2

The integral of two separate functions which are added together is the same as each function integrated separately then added together.

$$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

Example

$$\begin{aligned} & \int (5x^2 + 4e^x) dx \\ &= 5 \int x^2 dx + 4 \int e^x dx \\ &= \underline{\underline{\frac{5}{3}x^3 + 4e^x}} \end{aligned}$$

Rule #3

The addition of a constant to a variable doesn't change the form of the integral. However, x must be in the first degree ie no higher powers of x are involved. ('a' is a constant)

$$\text{if } \int f(x) dx = F(x) \text{ then } \int f(x+a) dx = F(x+a)$$

Example

$$\int x^3 dx = \frac{x^4}{4} \Rightarrow \int (x+5)^3 dx = \frac{(x+5)^4}{4}$$

Rule #4

If 'a' & 'b' are constants then x can be replaced by 'bx+a' with the integral remaining in the same form.

$$\text{if } \int f(x)dx = F(x)$$

$$\text{then } \int f(bx+a)dx = \frac{1}{b}F(bx+a)$$

Example

$$\int xdx = \frac{x^2}{2}$$

$$\begin{aligned} \int (3x+4)dx &= \frac{1}{3} \frac{(3x+4)^2}{2} \\ &= \frac{(3x+4)^2}{6} \end{aligned}$$
