Integration: By Substitution

The Substitution method(or 'changing the variable')

This is best explained with an example:

$$\int (2x+5)^3 \, dx$$

Like the Chain Rule simply make one part of the function equal to a variable eg u,v, t etc.

$$\text{let} \qquad t = (2x+5)$$

Differentiate the equation with respect to the chosen variable.

$$\frac{dt}{dx} = 2$$

Rearrange the substitution equation to make 'dx' the subject.

$$dx = \frac{dt}{2}$$

Substitute for 'dx' into the original expression.

$$\int (2x+5)^3 \frac{dt}{2}$$

Substitute the chosen variable into the original function.

$$\int t^3 . \frac{dt}{2}$$

Integrate with respect to the chosen variable.

$$\int \frac{t^3}{2} dt = \frac{t^4}{2.4} = \frac{t^4}{8} + C$$

Restate the original expression and substitute for t.

$$\int (2x+5)^3 dx = \frac{(2x+5)^4}{8} + C$$

NB Don't forget to add the Constant of Integration(C) at the end. Remember this is an indefinite integral.

Example #1

$$\int 5xe^{x^2+1}dx$$

make $\quad t = x^2 + 1$

then $\quad \frac{dt}{dx} = 2x$

$$xdx = \frac{dt}{2}$$

$$\int 5xe^{x^2+1}dx = \int 5e^t.\frac{dt}{2}$$

$$= \int \frac{5}{2}e^t dt$$

$$= \frac{5}{2}e^t + C$$

$$= \frac{5}{2}e^{x^2+1} + C$$

Example #2

$$\int \frac{1}{x}\ln x dx$$

make $\quad t = \ln x$

then $\quad \frac{dt}{dx} = \frac{1}{x}$

$$xdt = dx$$

$$dt = \frac{dx}{x}$$

$$\int \frac{1}{x}\ln x dx = \int t dt$$

$$= \frac{t^2}{2}$$

$$= \frac{1}{2}(\ln x)$$

Example #3

$$\int \sin^3 x \cos x dx$$

let $t = \sin x$

then $\dfrac{dt}{dx} = \cos x$

$$dt = \cos x dx$$

$$\int \sin^3 x \cos x dx = \int t^3 dt$$

$$= \frac{t^4}{4} + C$$

$$= \frac{\sin^4 x}{4} + C$$