Integration: By Parts

Integration 'By Parts' - from the Product Rule

The integration of expressions where there are two separate functions multiplied together, is essentially by an amended version of Leibnitz's Product Rule.

to integrate an expression of the type

$$\int f(x).g(x)dx$$

make $u = f(x)$ and $v = g(x)$

the expression becomes $\int uv dx$

using the Product Rule:

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\Rightarrow \qquad u\frac{dv}{dx} = \frac{d(uv)}{dx} - v\frac{du}{dx}$$

integrating with respect to x

$$\int u\left(\frac{dv}{dx}\right)dx = uv - \int v\left(\frac{du}{dx}\right)dx$$

$$\underline{\int u dv = uv - \int v du}$$

Example #1

integrate $\int 2xe^{3x}dx$

make $u = 2x$ and $\frac{dv}{dx} = e^{3x}$

$dv = e^{3x}dx$

then $\frac{du}{dx} = 2$ and $v = \frac{1}{3}e^{3x}$

$du = 2dx$

using the formula

$$\int udv = uv - \int vdu$$

$$\int 2xe^{3x}dx = 2x.\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}.2dx$$

$$= \frac{2x}{3}e^{3x} - \frac{2}{3}.\frac{1}{3}e^{3x}$$

$$= \frac{2x}{3}e^{3x} - \frac{2}{9}e^{3x} + C$$

Example #2

integrate $\int 2x\cos x dx$

make $u = 2x$ and $\frac{dv}{dx} = \cos x$

$dv = \cos x dx$

then $\frac{du}{dx} = 2$ and $v = \sin x$

$du = 2dx$

using the formula

$$\int udv = uv - \int vdu$$

$$\int 2x\cos x dx = 2x.\sin x - \int \sin x.2dx$$

$$= 2x\sin x - 2(-\cos x) + C$$

$$= 2x\sin x + 2\cos x + C$$

Example #3

integrate $\int 3x \ln x dx$

make $u = \ln x$ and $\frac{dv}{dx} = 3x$

$dv = 3x dx$

then $\frac{du}{dx} = \frac{1}{x}$ and $v = \frac{3}{2}x^2$

$du = \frac{dx}{x}$

using the formula

$$\int u dv = uv - \int v du$$

$$\int 3x \ln x dx = \ln x . \frac{3}{2}x^2 - \int \frac{3}{2}x^2 . \frac{dx}{x}$$

$$= \frac{3}{2}x^2 \ln x - \int \frac{3}{2} x dx$$

$$= \frac{3}{2}x^2 \ln x - \frac{3}{2} . \frac{x^2}{2}$$

$$= \frac{3}{2}x^2 \ln x - \frac{3x^2}{4}$$