Algebraic Fractions

Before beginning this topic it is advised that you read and understand 'partial fractions' in the **algebra** section first. The examples given here assume knowledge of this.

Denominator 1st degree(x)

Example

$$\text{find} \quad \int \frac{3}{5x+2}dx$$

$$\text{using} \int \frac{1}{ax+b} = \frac{1}{a}\ln|ax+b|$$

$$\Rightarrow \quad \int \frac{3}{5x+2}dx = 3.\frac{1}{5}\ln|5x+2|$$

$$\int \frac{3}{5x+2}dx = \frac{3}{5}\ln|5x+2|$$

Denominator 2nd degree(x^2)

Example

find $\int \frac{2x-2}{(x+1)^2}dx$

using partial fractions

$$\frac{2x-2}{(x+1)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\equiv \frac{A(x+1)+B}{(x+1)^2}$$

$$2x-2 = A(x+1)+B$$

making $x=-1$

$$-2-2 = \quad 0 \quad +B$$

$$\underline{B=-4}$$

making $x=2$

$$4-2 = A(3)-4$$

$$2 = 3A-4, \quad 3A=6, \quad \underline{A=2}$$

$$\Rightarrow \quad \frac{2x-2}{(x+1)^2} \equiv \frac{2}{x+1} - \frac{4}{(x+1)^2}$$

$$\int \frac{2x-2}{(x+1)^2}dx = \int \left(\frac{2}{x+1} - \frac{4}{(x+1)^2} \right) dx$$

$$\int \frac{2x-2}{(x+1)^2}dx = 2\ln(x+1) + 4(x+1)^{-1}$$

Denominator 3rd degree(x^3)

Example

find $\int \frac{5+x}{(1-x)(5+x^2)}dx$

using partial fractions

$$\frac{5+x}{(1-x)(5+x^2)} \equiv \frac{A}{x+1} + \frac{Bx+C}{5+x^2}$$

$$\equiv \frac{A(5+x^2)+(Bx+C)(1-x)}{(1-x)(5+x^2)}$$

$$5+x = A(5+x^2)+(Bx+C)(1-x)$$

$$5+x = 5A + Ax^2 + Bx + 1 - Bx^2 - cx \quad *$$

making $x = 1$

$$5+1 = A(5+1) + \quad 0$$

$$6 = 6A, \qquad \underline{A = 1}$$

making $x = 0$

$$5 = 5A + C, \qquad 5 = 5 + C, \quad \underline{C = 0}$$

equating coefficients of x^2 *

$$0 = A - B, \quad \underline{B = A = 1}$$

$$\Rightarrow \quad \frac{5+x}{(1-x)(5+x^2)} \equiv \frac{1}{1-x} + \frac{x}{5+x^2}$$

$$\int \frac{5+x}{(1-x)(5+x^2)}dx = \int \left(\frac{1}{1-x} - \frac{x}{5+x^2} \right) dx$$

$$\int \frac{5+x}{(1-x)(5+x^2)}dx = -\ln(x-1) + \frac{1}{2}\ln(5+x^2)$$
