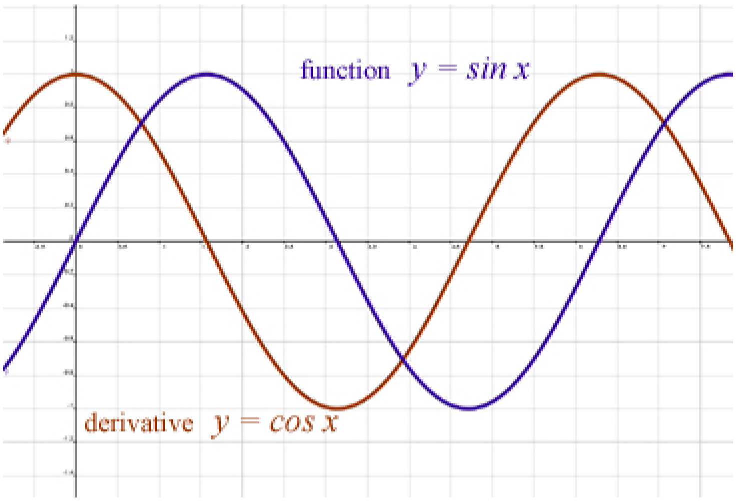
Calculus: Trigonometrical Functions

Relation between derived trigonometrical functions

$$\tan x = \frac{\sin x}{\cos x} \qquad \text{cotan } x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \text{cosec } x = \frac{1}{\sin x}$$

Derivative of the Sine Function



$$\frac{d(\sin x)}{dx} = \cos x$$

Example differentiate sin(2x+4)

$$\text{let } \ y = \sin(2x+4) \quad \text{and} \quad t = 2x+4$$

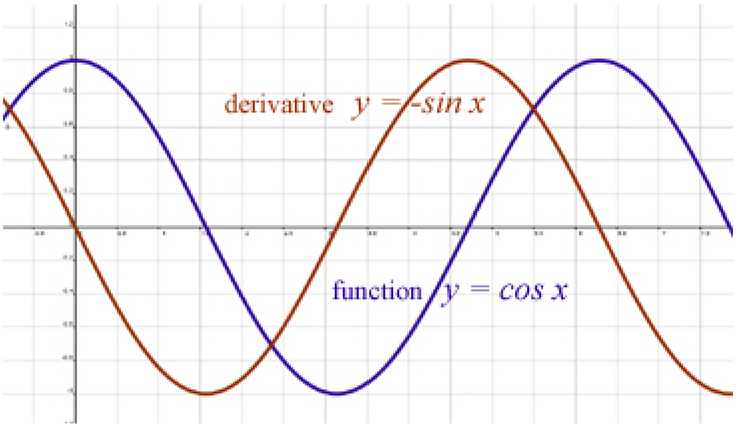
$$\therefore \ y = \sin(t)$$

$$\frac{dy}{dt} = \cos(t) \qquad \frac{dt}{dx} = 2$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \cos(t).2$$

$$= \underline{2\cos(2x+4)}$$

Derivative of the Cosine Function



$$\frac{d(\cos x)}{dx} = -\sin x$$

Example differentiate $\cos^3 x$

$$\text{let } y = \cos^3 x \quad \text{and} \quad t = \cos x$$

$$\therefore \quad y = t^3$$

$$\frac{dy}{dt} = 3t^2 \qquad \frac{dt}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 3t^2.(-\sin x)$$

$$= -3\cos^2 x \sin x$$

$$= -\frac{3}{2}\cos x(2\cos x \sin x)$$

$$\text{but} \quad \sin 2\theta = 2\cos\theta \sin\theta$$

$$\therefore \frac{dy}{dx} = -\frac{3}{2}\cos x \sin 2x$$

Derivative of the Tangent Function

$$\frac{d(\tan x)}{dx} = \frac{d\left(\dfrac{\sin x}{\cos x}\right)}{dx}$$

using the Product Rule

$$y = \frac{u}{v} \qquad u = \sin x \quad v = \cos x$$

$$\frac{du}{dx} = \cos x \qquad \frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x).(\cos x) - (\sin x).(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\underline{\frac{d(\tan x)}{dx} = \sec^2 x}$$

Derivative of the Cosecant Function

$$\frac{d(\operatorname{cosec} x)}{dx} = \frac{d\left(\dfrac{1}{\sin x}\right)}{dx} = \frac{d\left(\sin x\right)^{-1}}{dx}$$

let $y = \left(\sin x\right)^{-1}$, $t = \sin x$

then $y = t^{-1}$

$$\frac{dy}{dt} = (-1)(t^{-2}) \qquad \frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -t^{-2}.\cos x = -(\sin x)^{-2}.\cos x$$

$$\frac{dy}{dx} = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x.\sin x} = -\frac{\cos x}{\sin x}.\frac{1}{\sin x}$$

$$\underline{\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cotan} x.\operatorname{cosec} x}$$

Derivative of the Secant Function

$$\frac{d(\sec x)}{dx} = \frac{d\left(\frac{1}{\cos x}\right)}{dx} = \frac{d(\cos x)^{-1}}{dx}$$

let $y = (\cos x)^{-1}$, $t = \cos x$

then $y = t^{-1}$

$$\frac{dy}{dt} = (-1)(t^{-2}) \qquad \frac{dt}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -t^{-2}.(-\sin x) = (\cos x)^{-2}.\sin x$$

$$\frac{dy}{dx} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x.\cos x} = \frac{\sin x}{\cos x}.\frac{1}{\cos x}$$

$$\underline{\frac{d(\sec x)}{dx} = \tan x.\sec x}$$

Derivative of the Cotangent Function

$$\frac{d(\cot x)}{dx} = \frac{d\left(\frac{\cos x}{\sin x}\right)}{dx}$$

using the Product Rule

$$y = \frac{u}{v} \qquad u = \cos x \quad v = \sin x$$

$$\frac{du}{dx} = -\sin x \qquad \frac{dv}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(\sin x).(-\sin x) - (\cos x).(\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} = -\mathrm{cosec}^2 x$$

$$\underline{\frac{d(\cot x)}{dx} = -\mathrm{cosec}^2 x}$$