Calculus: the Quotient Rule

The Product Rule Equation

This is a variation on the Product Rule(**Leibniz's Law**) from the previous topic.

As with the Product Rule, , if **u** and **v** are two differentiable functions of *x*, then the differential of ***u / v*** is given by:

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

this can also be written, using '**prime notation**' as :

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

Example #1

$$\text{differentiate} \quad \frac{(x-3)^2}{(x+2)^2}$$

$$u = (x-3)^2 \qquad v = (x+2)^2$$

$$y = \frac{u}{v}$$

$$\frac{du}{dx} = 2(x-3) \qquad \frac{dv}{dx} = 2(x+2)$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(x+2)^2.2(x-3) - (x-3)^2.2(x+2)}{(x+2)^4}$$

$$= \frac{(x+2)^2.2(x-3) - (x-3)^2.2(x+2)}{(x+2)^4}$$

$$= \frac{2(x+2)(x-3)\{(x+2)-(x-3)\}}{(x+2)^4}$$

$$= \frac{2(x-3)\{x+2-x+3\}}{(x+2)^3}$$

$$= \frac{2(x-3)(5)}{(x+2)^3}$$

$$\underline{\frac{dy}{dx} = \frac{10(x-3)}{(x+2)^3}}$$

Example #2

differentiate $\dfrac{x}{\sqrt{(1+x^2)}}$

$u = x \qquad v = (1+x^2)^{1/2}$

$$y = \frac{u}{v}$$

$$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2}.2x(1+x^2)^{-1/2}$$

$$= x(1+x^2)^{-1/2}$$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{(1+x^2)^{1/2}.1 - x.x(1+x^2)^{-1/2}}{(1+x^2)}$$

$$= \frac{(1+x^2)^{1/2} - x^2(1+x^2)^{-1/2}}{(1+x^2)}$$

mult. top & bottom by $(1+x^2)^{1/2}$

$$= \frac{(1+x^2) - x^2}{(1+x^2)^{3/2}}$$

$$\underline{\frac{dy}{dx} = \frac{1}{(1+x^2)^{3/2}}}$$

Example #3

differentiate $\dfrac{1-x^2}{1+x^2}$

$u = 1 - x^2 \qquad v = 1 + x^2$

$$y = \frac{u}{v}$$

$\dfrac{du}{dx} = -2x \qquad \dfrac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{(1+x^2).(-2x) - (1-x^2).2x}{(1+x^2)^2}$$

$$= \frac{-(2x+2x^3) - (2x-2x^3)}{(1+x^2)^2}$$

$$= \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2}$$

$$\frac{dy}{dx} = -\frac{4x}{(1+x^2)^2}$$