

## Calculus: the Product Rule

### The Product Rule Equation

**Gottfried Leibniz** is credited with the discovery of this rule which he called **Leibniz's Law**.

Simply, if **u** and **v** are two differentiable functions of  $x$ , then the differential of **uv** is given by:

$$y = uv$$
$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

this can also be written, using '**prime notation**' as :

$$(u.v)' = u.v' + vu'$$

Example #1differentiate  $(x^2 + 1)^3(x^3 + 1)^2$ 

$$u = (x^2 + 1)^3 \quad v = (x^3 + 1)^2$$

$$y = u.v$$

$$\frac{du}{dx} = 3(x^2 + 1)^2 \cdot 2x \quad \frac{dv}{dx} = 2(x^3 + 1) \cdot 3x^2$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= (x^2 + 1)^3 \cdot 2(x^3 + 1) \cdot 3x^2 + (x^3 + 1)^2 \cdot 3(x^2 + 1)^2 \cdot 2x \\ &= 6x^2(x^2 + 1)^3(x^3 + 1) + 6x(x^2 + 1)^2(x^3 + 1)^2 \\ &= 6x(x^2 + 1)^2(x^3 + 1) \{x(x^2 + 1) + (x^3 + 1)\} \\ &= 6x(x^2 + 1)^2(x^3 + 1) \{x^3 + x + x^3 + 1\} \\ &= \underline{6x(x^2 + 1)^2(x^3 + 1)(2x^3 + x + 1)} \end{aligned}$$

Example #2differentiate  $y = (x^2 - 4)(x + 3)^2$ 

$$u = (x^2 - 4) \quad v = (x + 3)^2$$

$$y = u.v$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = 2(x + 3)$$

$$\text{using } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= (x^2 - 4) \cdot 2(x + 3) + (x + 3)^2 \cdot 2x \\ &= 2(x + 3)(x^2 - 4) + 2x(x + 3)^2 \\ &= 2(x + 3) \{x^2 - 4 + x^2 + 3x\} \\ &= \underline{2(x + 3)(2x^2 + 3x - 4)} \end{aligned}$$

Example #3

differentiate    $y = (x^2 + 3)\sqrt{(2+x)}$

$$u = (x^2 + 3) \quad v = (2+x)^{\frac{1}{2}}$$

$$y = u.v$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = \frac{1}{2}(2+x)^{-\frac{1}{2}}$$

using    $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

$$\frac{dy}{dx} = (x^2 + 3) \cdot \frac{1}{2}(2+x)^{-\frac{1}{2}} + (2+x)^{\frac{1}{2}} \cdot 2x$$

$$= \frac{(x^2 + 3)}{2(2+x)^{\frac{1}{2}}} + \frac{2x(2+x)^{\frac{1}{2}}}{1}$$

$$= \frac{(x^2 + 3) + 2(2+x)^{\frac{1}{2}} \cdot 2x(2+x)^{\frac{1}{2}}}{2(2+x)^{\frac{1}{2}}}$$

$$= \frac{(x^2 + 3) + 4x(2+x)}{2(2+x)^{\frac{1}{2}}}$$

$$= \frac{(x^2 + 3) + 8x + 4x^2}{2(2+x)^{\frac{1}{2}}}$$

$$= \frac{5x^2 + 8x + 3}{2(2+x)^{\frac{1}{2}}}$$

$$\underline{\underline{\frac{dy}{dx} = \frac{(5x+3)(x+1)}{2(2+x)^{\frac{1}{2}}}}}$$