Calculus: Parametric Equations

Parametric Equations

Both x and y are given as functions of another variable - called a **parameter** (eg 't'). Thus a pair of equations, called **parametric** equations, completely describe a single x-y function.

The d ifferentiation of functions given in parametric form is carried out using the Chain Rule.

Example #1

$$\text{find } \frac{dy}{dx} \text{ when } \quad x = t^2 , \quad y = 2t$$

$$x = t^2 \qquad\qquad y = 2t$$

$$\frac{dx}{dt} = 2t \qquad\qquad \frac{dy}{dt} = 2$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 2.\frac{1}{2t}$$

$$\underline{\frac{dy}{dx} = \frac{1}{t}}$$

Example #2

given that $x = 3\cos\theta - \cos 3\theta$, $y = 3\sin\theta - \sin 3\theta$

show that $\dfrac{dy}{dx} = \tan 2\theta$

$$x = 3\cos\theta - \cos 3\theta \qquad y = 3\sin\theta - \sin 3\theta$$

$$\frac{dx}{d\theta} = -3\sin\theta + 3\sin 3\theta\ , \quad \frac{dy}{d\theta} = 3\cos\theta - 3\cos 3\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}.\frac{dx}{d\theta} = 3\cos\theta - 3\cos 3\theta.\frac{1}{-3\sin\theta + 3\sin 3\theta}$$

$$\frac{dy}{dx} = \frac{3\cos\theta - 3\cos 3\theta}{3\sin 3\theta - 3\sin\theta} = \frac{\cos\theta - \cos 3\theta}{\sin 3\theta - \sin\theta}$$

but $\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

$\therefore \quad \cos(2\theta+\theta) - \cos(2\theta-\theta) = -2\sin 2\theta\sin\theta$

$\Rightarrow \quad \cos(3\theta) - \cos(\theta) = -2\sin 2\theta\sin\theta$

$\Rightarrow \quad \cos(\theta) - \cos(3\theta) = 2\sin 2\theta\sin\theta$

also $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

$\therefore \quad \sin(2\theta+\theta) - \sin(2\theta-\theta) = 2\cos 2\theta\sin\theta$

$\Rightarrow \quad \sin(3\theta) - \sin(\theta) = 2\cos\theta\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{2\sin 2\theta\sin\theta}{2\cos 2\theta\sin\theta}$$

$$= \underline{\tan 2\theta}$$

Example #3

if $x = t^3 - t^2$ and $y = t^2 - t$,

find $\dfrac{dy}{dx}$ in terms of t

$$x = t^3 - t^2 \quad \therefore \quad \frac{dx}{dt} = 3t^2 - 2t$$

$$y = t^2 - t \quad \therefore \quad \frac{dy}{dt} = 2t - 1$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dt}.\frac{dt}{dx} \\ &= 2t - 1.\frac{1}{3t^2 - 2} \\ &= \underline{\frac{2t - 1}{t(3t - 2)}}\end{aligned}$$