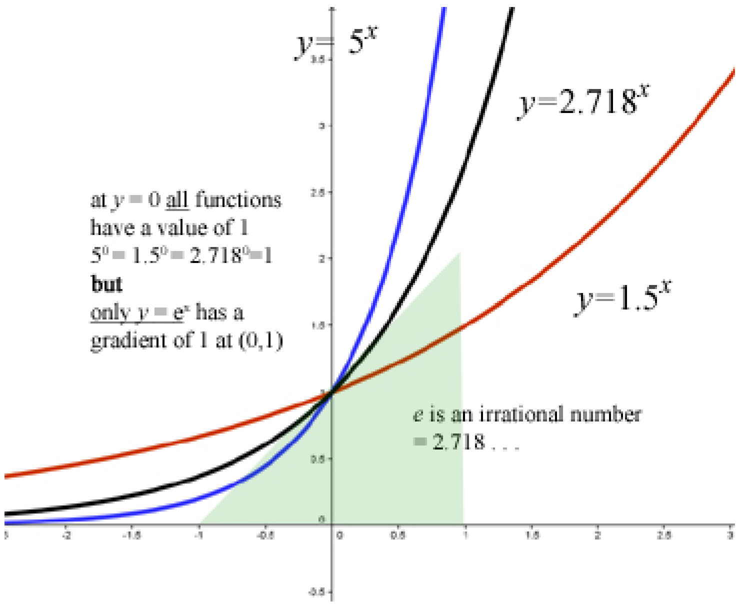
Calculus: dy/dx Exponentials, Logarithms

Exponential functions

Strictly speaking **all** functions where the variable is in the index are called exponentials.

The Exponential function e^x

This is the **one** particular exponential function where 'e' is approximately 2.71828 and the gradient of $y = e^x$ at (0,1) is 1.



One other special quality of $y = e^x$ is that its derivative is also equal to e^x

$$\frac{d(e^x)}{dx} = e^x$$

and for problems of the type $y = e^{kx}$

$$t = kx \qquad \frac{dt}{dx} = k$$

$$y = e^t \qquad \frac{dy}{dt} = e^t = e^{kx}$$

$$\frac{d(e^{kx})}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = e^{kx}.k$$

$$\underline{\frac{dy}{dx} = ke^{kx}}$$

Derivative problems like the above concerning 'e' are commonly solved using the Chain Rule.

Example #1

Find the derivative of:

$$y = e^{2x^3}$$

$$\text{let } t = 2x^3, \qquad y = e^t$$

$$\frac{dx}{dt} = 6x^2 \qquad \frac{dy}{dt} = e^t$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dx}{dt} = e^t.6x^2$$

$$\underline{\frac{dy}{dx} = e^{2x^3}6x^2}$$

Example #2

find the derivative of:

$$y = e^{(3x-4)^2}$$

$$\text{let} \quad t = (3x-4)^2 \qquad y = e^t$$

$$\frac{dt}{dx} = 2(3x-4).3 \ , \qquad \frac{dy}{dt} = e^t$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = e^t.6(3x-4)$$

$$\frac{dy}{dx} = \underline{6(3x-4)e^{(3x-4)^2}}$$

Derivative of a Natural Logarithm function

Remember $\mathbf{y=\log_e x}$ means:

x is the number produced when **e** is raised to the power of **y**

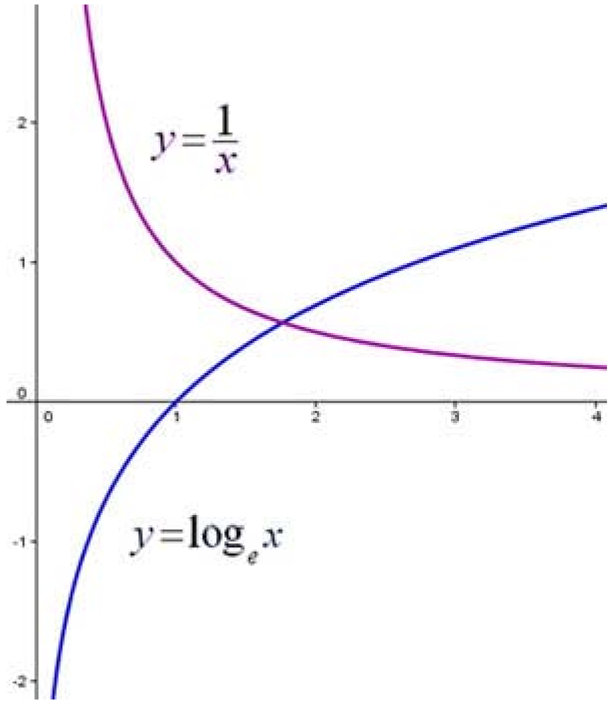
The connection between $y=e^x$ and $y=\log_e x$ can be shown by rearranging $y=\log_e x$.

$\mathbf{y=\log_e x}$ can be written as $\mathbf{x=e^y}$

($\mathbf{\log_e x}$ is now more commonly written as **ln(x)**)

The derivative of ln(x) is given by:

$$\frac{d(\ln(x))}{dx} = \frac{1}{x}$$



Example #1

find the derivative of y = ln(3x)

$$y = \ln(3x)$$
$$= \ln(3) + \ln(x)$$
$$\frac{dy}{dx} = 0 + \frac{1}{x}$$
$$\frac{dy}{dx} = \frac{1}{x}$$

Example #2

find the derivative of y = ln(x^3+3)

$$y = \ln(x^2+3)$$

$$\text{let} \quad t = x^2+3, \quad \text{then} \quad y = \ln(t)$$

$$\frac{dt}{dx} = 2x \ , \qquad \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{1}{t}.2x = \frac{1}{x^2+3}.2x$$

$$\underline{\frac{dy}{dx} = \frac{2x}{x^2+3}}$$

Problems of the type $y=N^{f(x)}$

Problems of this type are solved by taking logs on both sides and/or using the Chain Rule.

Example #1

find the derivative of $y=10^x$

$$y = 10^x$$

$$\ln(y) = \ln(10^x)$$

$$\ln(y) = x\ln(10)$$

$$\frac{d(\ln(y))}{dx} = \frac{d(x\ln(10))}{dx}$$

$$\frac{d(\ln(y)}{dy}.\frac{dy}{dx} = 1.\ln(10)$$

$$\frac{1}{y}.\frac{dy}{dx} = \ln(10)$$

$$\frac{dy}{dx} = y\ln(10) = \underline{10^x\ln(10)}$$

Example #2

find the derivative of y= ln($\cos^3$2x)

$$y = \ln(\cos^3 2x)$$

$$t = \cos(2x) \qquad y = \ln(t^3)$$

$$\frac{dt}{dx} = -2\sin(2x) \qquad \frac{dy}{dt} = \frac{1}{t^3}.3t^2 = \frac{3}{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{3}{t}.(-2\sin(2x))$$

$$= \frac{3}{\cos(2x)}.(-2\sin(2x))$$

$$= \underline{-6\tan(2x)}$$

A graphical comparison of exponential and log functions

As you can see, y= e^x is reflected in the line y=x to produce the curve y=ln(x)