Integration: Definite Integrals

The 'definite Integral' equation

If a function F(x) is the integral of the function f(x)

$$F(x) = \int f(x)dx$$

then an integral of the form:

$$\int_a^b f(x)dx = \left[F(x) \right]_a^b = F(b) - F(a)$$

is known as the **definite integral**, where a, b are called the **limits** of the integral.

Example #1

$$\text{evaluate } \int_1^2 (2x^2 + 1)dx$$

$$\int_1^2 (2x^2 + 1)dx = \left[\frac{2x^3}{3} + x \right]_1^2$$

$$= \left[\frac{2.2^3}{3} + 2 \right] - \left[\frac{2.1^3}{3} + 1 \right]$$

$$= \left[\frac{2.8}{3} + 2 \right] - \left[\frac{2}{3} + 1 \right]$$

$$= \frac{16}{3} + 2 - \frac{2}{3} + 1$$

$$= \frac{14}{3} + 3 = 4\tfrac{2}{3} + 3 = 7\tfrac{2}{3}$$

$$\underline{\int_1^2 (2x^2 + 1)dx = 7\tfrac{2}{3}}$$

Example #2

evaluate $\int_2^3 (x^3 + 2x)dx$

$$\int_2^3 (x^3 + 2x)dx = \left[\frac{x^4}{4} + x^2\right]_2^3$$
$$= \left[\frac{3^4}{4} + 3^2\right] - \left[\frac{2^4}{4} + 2^2\right]$$
$$= \left[\frac{81}{4} + 9\right] - \left[\frac{16}{4} + 4\right]$$
$$= 20\tfrac{1}{4} + 9 - 4 - 4$$
$$= 21\tfrac{1}{4}$$

$$\underline{\int_2^3 (x^3 + 2x)dx = 21\tfrac{1}{4}}$$

Example #3

evaluate $\int_0^2 (3x^2 + 2x + 5)dx$

$$\int_0^2 (3x^2 + 2x + 5)dx = \left[\frac{3x^3}{3} + \frac{2x^2}{2} + 5x\right]_0^2$$
$$= \left[2^3 + 2^2 + 5.2^1\right] - \left[0\right]$$
$$= \left[8 + 4 + 10\right]$$
$$= 22$$

$$\underline{\int_0^2 (3x^2 + 2x + 5)dx = 22}$$