Compound Angles

The Six Compound Angle Identities

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A+B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Angle Formulae

making $A = B$,

$$\sin 2A = 2(\sin A \cos A)$$
$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = \cos^2 A - \sin^2 A \qquad \text{(i}$$

but $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \quad \cos^2 A = 1 - \sin^2 A$

$\Rightarrow \quad \sin^2 A = 1 - \cos^2 A$

substituting for $\cos^2 A$ in (i

$$\cos 2A = 1 - \sin^2 A - \sin^2 A$$
$$\underline{\cos 2A = 1 - 2\sin^2 A}$$

substituting for $\sin^2 A$ in (i

$$\cos 2A = \cos^2 A - (1 - \cos^2 A)$$
$$\cos 2A = \cos^2 A - 1 + \cos^2 A)$$
$$\underline{\cos 2A = 2\cos^2 A - 1}$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Factor Formulae

$$\cos(A+B)+\cos(A-B)=2\cos A\cos B$$
$$\cos(A+B)-\cos(A-B)=-2\sin A\sin B$$
$$\sin(A+B)+\sin(A-B)=2\sin A\cos B$$
$$\sin(A+B)-\sin(A-B)=2\cos A\sin B$$

let $P=A+B$ and $Q=A-B$

adding $P+Q=2A$

$\therefore$ $\underline{A=\dfrac{P+Q}{2}}$

subtracting $P-Q=2B$

$\therefore$ $\underline{B=\dfrac{P-Q}{2}}$

substituting for A, B above

$$\cos P+\cos Q=2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$
$$\cos P-\cos Q=-2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$$
$$\sin P+\sin Q=2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$
$$\sin P-\sin Q=2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

rcos() form

Adding a sine and a cosine will generate a cosine curve. This will have a larger amplitude than the original and is out of phase with it.

Writing the expression as **rcos(θ - α)**,

'**r**' amplitude
'**α**' no. degrees phase difference(to the right)

Thus expressions of the form:

$$a\cos\theta + b\sin\theta$$

can be rewritten as

$$r\cos(\theta + \alpha)$$

Finding 'r' and the phase angle 'α'

$$r\cos(\theta+\alpha) = r\ \cos\theta\ \cos\alpha - r\ \sin\theta\ \sin\alpha$$
$$= r\ \cos\alpha\ \cos\theta - r\ \sin\alpha\ \sin\theta$$

let

$a = r\cos\alpha$ (i

$b = -r\sin\alpha$ (ii

so that

$(r\ \cos\alpha)\ \cos\theta + (-r\ \sin\alpha)\ \sin\theta = a\cos\theta\ +\ b\sin\theta$

α is found by dividing (ii by (i

$$\frac{b}{a} = \frac{-r\sin\alpha}{r\cos\alpha}$$

$$\Rightarrow \qquad \underline{\tan\alpha = -\frac{b}{a}}$$

r is found by squaring and then adding (i & (ii

$a^2 + b^2 = r^2\cos^2\alpha + r^2\sin^2\alpha$

$a^2 + b^2 = r^2(\cos^2\alpha + \sin^2\alpha)$

but $\cos^2\alpha + \sin^2\alpha = 1$

$\therefore \qquad a^2 + b^2 = r^2$

$$\underline{r = \sqrt{a^2 + b^2}}$$

n.b. $a\cos\theta\ +\ b\sin\theta$ can also be written using the sine expression $r\sin(\theta+\alpha)$