Parametric Equations

Introduction

There is another way of writing y as a function of x and that is to use two separate equations.

One equation has x as a function of t (or θ) eg $x = 2t$

and the other equation has y as function of t (or θ) eg $y = t^2$

The variables t, θ are called **parameters** and the two equalities **parametric equations**.

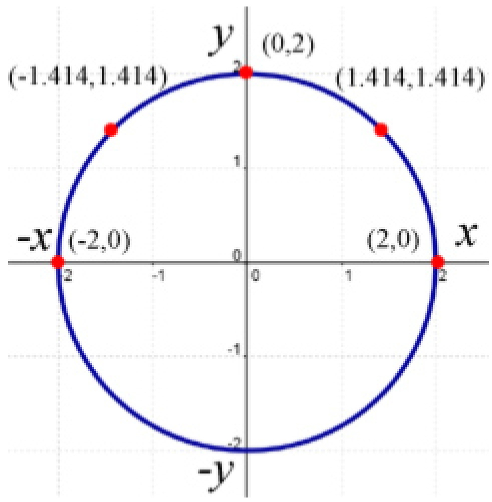
Common questions on this topic are the plotting of parametric equations and their conversion to a single Cartesian equation.

Example #1

Plot the graph of the curve given parametrically by the equations:

$$x = 2\cos\theta \qquad y = 2\sin\theta$$

θ°	0	45	90	135	180
x = 2cosθ	2	1.414	0	-1.414	-2
y = 2sinθ	0	1.414	2	1.414	0



Example #2

What is the Cartesian equation given parametrically by:

$x = t^2+3 \qquad y = t^3 + 3t$

$x = t^2 + 3 \qquad$ (i

$y = t^3 + 3t \qquad$ (ii

factorising (ii $\qquad y = t(t^2 + 3)$

but $\quad x = t^2 + 3$

$\therefore \quad y = tx, \quad \Rightarrow \quad t = \dfrac{y}{x}$

substituting for t in (i

$$x = \left(\frac{y}{x}\right)^2 + 3$$

$$x = \frac{y^2}{x^2} + 3$$

multiplying both sides by x^2

$$x^3 = y^2 + 3x^2$$

$$x^3 - 3x^2 = y^2$$

$$y^2 = x^3 - 3x^2$$

$$\underline{y^2 = x^2(x-3)}$$

Example #3

What is the Cartesian equation given parametrically by:

$x = 2\sin\theta \qquad y = 2\sin 2\theta$

$$y = 2\sin 2\theta \quad \text{but} \quad \sin 2\theta = 2\sin\theta\cos\theta$$

$$\therefore \quad y = 2(2\sin\theta\cos\theta)$$

$$y = 4\sin\theta\cos\theta$$

$$\therefore \quad y^2 = 16\sin^2\theta\cos^2\theta \qquad \text{(i}$$

$$\text{but} \quad x = 2\sin\theta \quad \Rightarrow \quad \frac{x}{2} = \sin\theta \qquad \text{(ii}$$

using the identity $\cos^2\theta = 1 - \sin^2\theta$

and substiuting for $\sin\theta$ from (ii

$$\Rightarrow \quad \cos^2\theta = 1 - \frac{x^2}{4}$$

now substituting in (i for $\sin\theta$ and $\cos^2\theta$

$$y^2 = 16\sin^2\theta\cos^2\theta$$

$$= 16\left(\frac{x}{2}\right)^2\left(1 - \frac{x^2}{4}\right)$$

$$= 16\frac{x^2}{4}\left(1 - \frac{x^2}{4}\right)$$

$$\underline{y^2 = 4x^2\left(1 - \frac{x^2}{4}\right)}$$