Arithmetical Series

Arithmetical series structure

An arithmetical series starts with the first term, usually given the letter '**a**'. For each subsequent term of the series another term is added. This is a multiple of the letter '**d**' called 'the **common difference**'.

So the series has the structure:

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \ldots + (l-d) + l$$

where S_n is the sum to '**n**' terms, the letter '**l**' is the last term.

The common difference 'd' is calculated by subtracting any term from the subsequent term.

The **nth term**(sometimes called the 'general term')is given by:

$$a + (n-1)d$$

Proof of the sum of an arithmetical series

The sum of an arithmetical series is found by adding two identical series together, but putting the second series in reverse order.

$$S_n = a + (a+d) + (a+2d) + \ldots + (l-2d) + (l-d) + l$$
$$S_n = l + (l-d) + (l-2d) + \ldots + (a+2d) + (a+d) + a$$
$$2S_n = (a+l) + (a+l) + (a+l) + \ldots + (a+l) + (a+l) + (a+l)$$

noting that there are still n terms on the RHS

$$\therefore\ 2S_n = n(a+l)$$

$$\Rightarrow \quad S_n = \frac{n(a+l)}{2} \qquad \text{(i}$$

since the last term l is given by:

$$l = a + (n-1)d$$

substituting for l in (i

$$S_n = \frac{n(a + a + (n-1)d)}{2}$$

$$\therefore \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

Example #1

In an arithmetical progression the 8th term is 23 and the 11th term is 4 times the 3rd term.

Find the 1st term, the common difference and the sum of the first 10 terms.

let the 1st term be a and the common difference d

then the 8th terms is given by:

$$a + 7d = 23 \qquad \text{(i}$$

the 11th term is $(a+10d)$, the 3rd term is $(a+2d)$

$$\therefore \quad a + 10d = 4(a+2d)$$

$$a + 10d = 4a + 8d$$

$$10d - 8d = 4a - a$$

$$2d = 3a, \quad \underline{a = \tfrac{2}{3}d} \qquad \text{(ii}$$

substituting into (i above for a

$$a + 7d = 23$$

$$\tfrac{2}{3}d + 7d = 23$$

$$2d + 21d = 69$$

$$23d = 69, \quad \underline{d = 3}$$

substituting into (ii for d

$$a = \tfrac{2}{3}(3), \quad \underline{a = 2}$$

$\therefore$ the first term is 2 and the common difference is 3

sum of the first 10 terms is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2(2) + (10-1)3)$$

$$= 5(4 + 9 \times 3)$$

$$= 5 \times 31$$

$$= 155$$

sum of the first 10 terms is 155

Example #2

The sum of terms of an arithmetic progression is 48.

If the first term is 3 and the common difference is 2, find the number of terms.

since

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

and $a = 3,\quad d = 2,\quad S_n = 48$

the sum of the first n terms is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$48 = \frac{n}{2}(2 \times 3 + (n-1)2)$$

$$96 = 6n + 2n^2 - 2n$$

$$2n^2 + 4n - 96 = 0$$

$$n^2 + 2n - 48 = 0$$

$$(n-6)(n+8) = 0$$

hence $n = 6$ or $n = -8$

$\therefore$ the number of terms is 6

Arithmetic Mean

This is a method of finding a term sandwiched between two other terms. The required term is calculated by taking an average of the term before and the term after.

So if we have a sequence of terms: **a b c** and **a** and **c** are known. Then term **b** is given by:

$$b = \frac{a+c}{2}$$

Example

If the 3rd term of an arithmetical progression is 7 and the 5th term is 11, what is the 4th term?

let the 3rd term be 'a' and the 5th term 'c'

then the 4th term 'b' is given by:

$$b = \frac{a+c}{2}$$

$$= \frac{7+11}{2}$$

$$= \frac{18}{2}$$

$$\underline{b = 9}$$