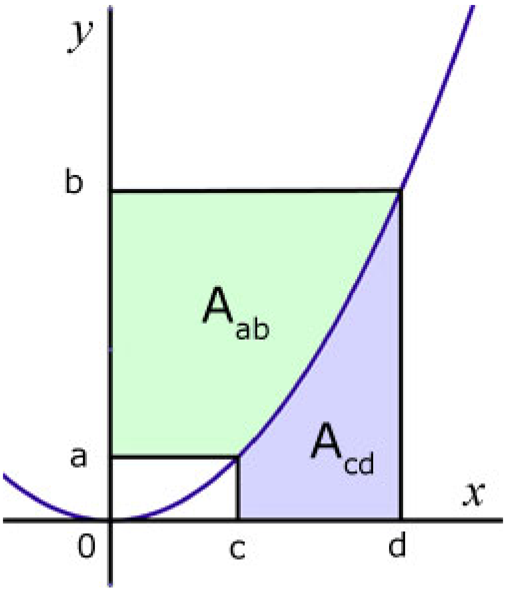
Integration: Area under a curve

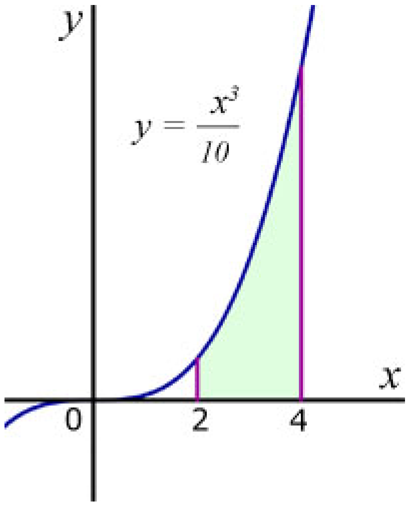
Area under a curve related to different axes



$$A_{ab} = \int_a^b x\,dy \qquad A_{cd} = \int_c^d y\,dx$$

Example #1

Find the area 'A' enclosed by the x-axis, x=2, x=4 and the graph of $y=x^3/10$.



$$A = \int_2^4 y\,dx$$

$$= \int_2^4 \frac{x^3}{10}\,dx$$

$$= \left[\frac{x^4}{4.10}\right]_2^4 = \left[\frac{x^4}{40}\right]_2^4$$

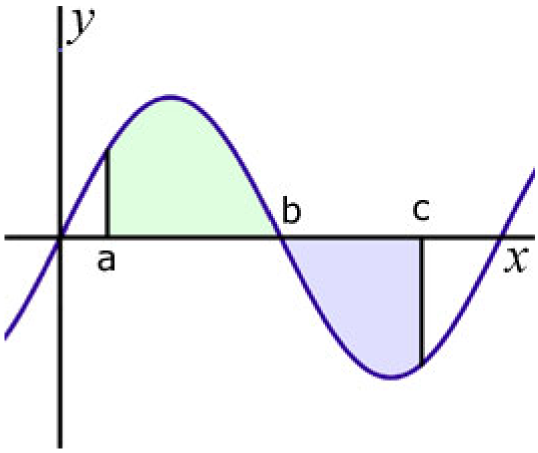
$$= \left[\frac{4^4}{40}\right] - \left[\frac{2^4}{40}\right]$$

$$= \left[\frac{256}{40}\right] - \left[\frac{16}{40}\right]$$

$$= \left[\frac{240}{40}\right] = [6]$$

area 'A' is 6

Positive and negative area



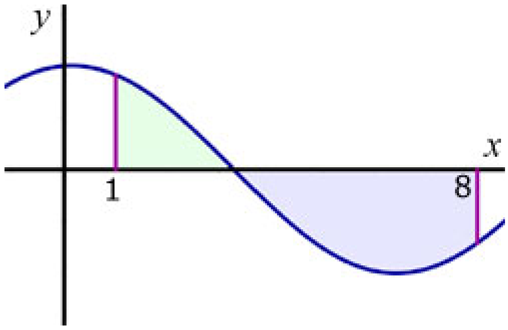
$$A_{ac} = \int_a^b y\,dx - \int_b^c y\,dx$$
$$= \int_a^b f(x)\,dx + \left|\int_b^c f(x)\,dx\right|$$

note: This expression calculates the **absolute area** between the curve the vertical lines at 'a' and 'b' and the x-axis. It takes no account of sign. If sign were an issue then the two integrals on the first line would be added and not subtracted.

Unless told differently, assume that the absolute area is required.

Example #1

Find the area 'A' enclosed by the x-axis, x=1, x=8 and the graph of y=2sin[(x+3)/2].



The curve crosses the x-axis at y=0.

Therefore 2sin[(x+3)/2]=0

Sine is zero when the angle is 0,180 or 360 deg.

(zero, pi and 2 pi)

$$\frac{x+3}{2}=\pi, \quad x+3=2\pi, \quad x=2\pi-3, \quad x=3.28$$

$$A=\int_{1}^{3.28} y\,dx+\left|\int_{3.28}^{4} y\,dx\right|$$

$$=\int_{1}^{3.28} 2\sin[(x+3)/2]dx+\left|\int_{3.28}^{8} 2\sin[(x+3)/2]dx\right|$$

$$A = \left[-2\cos\left(\frac{x+3}{2}\right)\right]_1^{3.28} + \left|\left[-2\cos\left(\frac{x+3}{2}\right)\right]_{3.28}^{8}\right|$$

$$= \left[-2\cos\left(\frac{3.28+3}{2}\right)\right] - \left[-2\cos\left(\frac{1+3}{2}\right)\right]$$
$$+ \left|\left[-2\cos\left(\frac{8+3}{2}\right)\right] - \left[-2\cos\left(\frac{3.28+3}{2}\right)\right]\right|$$

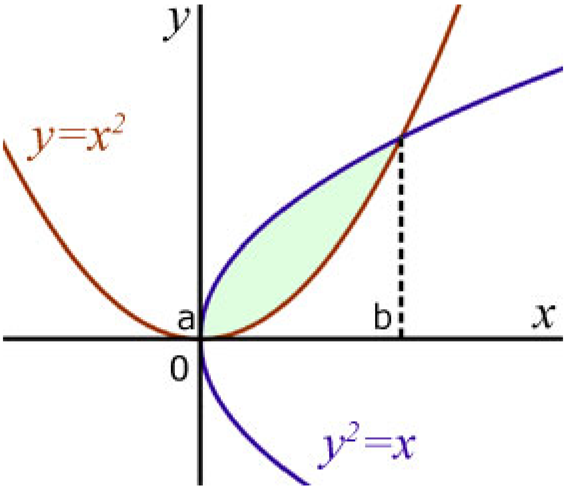
$$= \left[-2\cos(3.14)\right] - \left[-2\cos(2)\right]$$
$$+ \left|\left[-2\cos(5.5)\right] - \left[-2\cos(3.14)\right]\right|$$

$$= \left[-2(1)\right] - \left[-2(-0.416)\right]$$
$$+ \left|\left[-2(0.709)\right] - \left[-2(-1)\right]\right|$$

$$= 2 - 0.83 + \left|(-1.42 - 2)\right|$$
$$= 1.17 + 3.42$$
$$= 4.59$$

<u>area $A = 4.59$</u>

Area bounded by two curves

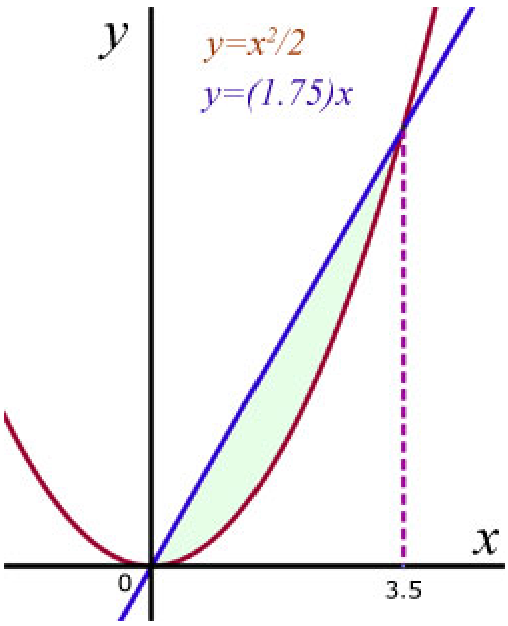


$$A_{ab} = \int_a^b y_1 dx - \int_a^b y_2 dx$$

Example

To 3 d.p. calculate the area 'A' included between the curves $y=x^2/2$ and $y=(0.75)x$

first find the x value where the curves cross



$$y_1 = \frac{x^2}{2}, \qquad y_2 = \frac{7x}{4}$$

where the curves cross $\quad y_1 = y_2$

$$\frac{x^2}{2} = \frac{7x}{4}, \quad \Rightarrow \quad 4x^2 = 14x$$

$$\Rightarrow \quad 2x = 7, \quad \therefore \quad \underline{x = 3.5}$$

The area 'A' is the difference between the area under the straight line and the area under the parabola, from x=0 to x=3.5 .

$$A = \int_0^{3.5} \frac{7}{4} x\,dx - \int_0^{3.5} \frac{x^2}{2} dx$$

$$= \left[\frac{7}{4}.\frac{x^2}{2}\right]_0^{3.5} - \left[\frac{1}{2}.\frac{x^3}{3}\right]_0^{3.5}$$

$$= \left[\frac{7}{8}.(3.5)^2\right] - \left[\frac{(3.5)^3}{6}\right]$$

$$= \frac{7}{8}(12.25) - \frac{1}{6}(42.875)$$

$$= 10.719 - 7.146$$

$$= 3.573$$

<u>area $A = 3.573$</u>