

Algebra : SurdsRules

Surds are mathematical expressions containing square roots. However, it must be emphasized that the square roots are 'irrational' i.e. they do not result in a whole number, a terminating decimal or a recurring decimal.

The rules governing surds are taken from the Laws of Indices.

rule #1

$$\sqrt{c} \times \sqrt{d} = \sqrt{c \times d}$$

$$c^{1/2} \times d^{1/2} = (c \times d)^{1/2}$$

examples

$$\sqrt{5} \times \sqrt{3} = \sqrt{3 \times 5} = \sqrt{15}$$

$$\sqrt{12} \times \sqrt{5} = \sqrt{12 \times 5} = \sqrt{60}$$

$$\sqrt{7} \times \sqrt{2} = \sqrt{7 \times 2} = \sqrt{14}$$

rule #2

$$\frac{\sqrt{c}}{\sqrt{d}} = \sqrt{\frac{c}{d}}$$

$$\frac{c^{1/2}}{d^{1/2}} = \left(\frac{c}{d}\right)^{1/2}$$

examples

$$\frac{\sqrt{15}}{\sqrt{5}} = \sqrt{\frac{15}{5}} = \sqrt{3}$$

$$\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}} = \sqrt{4} = 2$$

$$\frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}$$

Some Useful Expressions

expression #1

$$\begin{aligned}(c+d)^2 &= (c+d)(c+d) \\ &= c^2 + cd + cd + d^2 \\ &= c^2 + 2cd + d^2\end{aligned}$$

$$\begin{aligned}(c+\sqrt{d})^2 &= (c+\sqrt{d})(c+\sqrt{d}) \\ &= c^2 + c\sqrt{d} + c\sqrt{d} + d \\ &= c^2 + 2c\sqrt{d} + d\end{aligned}$$

$$\begin{aligned}(5+\sqrt{2})^2 &= (5+\sqrt{2})(5+\sqrt{2}) \\ &= 5^2 + 5\sqrt{2} + 5\sqrt{2} + 2 \\ &= 5^2 + 10\sqrt{2} + 2 \\ &= 25 + 10\sqrt{2} + 2 \\ &= 27 + 10\sqrt{2}\end{aligned}$$

expression #2 - (the difference of two squares)

$$\begin{aligned}(c+d)(c-d) &= c^2 + cd - cd + d^2 \\ &= c^2 - d^2\end{aligned}$$

$$\begin{aligned}(c+\sqrt{d})(c-\sqrt{d}) &= c^2 + c\sqrt{d} - c\sqrt{d} - (\sqrt{d})^2 \\ &= c^2 - d\end{aligned}$$

$$\begin{aligned}(7+\sqrt{3})(7-\sqrt{3}) &= 7^2 - (\sqrt{3})^2 \\ &= 49 - 3 \\ &= 46\end{aligned}$$

Rationalising Surds - This is a way of modifying surd expressions so that the square root is in the numerator of a fraction and not in the denominator.

The method is to multiply the top and bottom of the fraction by the square root.

$$\frac{5}{\sqrt{3}}, \quad \frac{5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{5 \times \sqrt{3}}{3}$$

$$\frac{8}{\sqrt{5}}, \quad \frac{8 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{8 \times \sqrt{5}}{5}$$

$$\frac{7}{\sqrt{2}}, \quad \frac{7 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{7 \times \sqrt{2}}{2}$$

Rationalising expressions using 'difference of two squares'

Remembering that : $(c + \sqrt{d})(c - \sqrt{d}) = c^2 - d$ from 'useful expressions' above.

Example #1 - simplify $\frac{2 + \sqrt{5}}{2 - \sqrt{5}}$

multiplying top and bottom by $(2 + \sqrt{5})$

$$\begin{aligned} & \frac{(2 + \sqrt{5})(2 + \sqrt{5})}{(2 - \sqrt{5})(2 + \sqrt{5})} \\ &= \frac{2^2 + 2\sqrt{5} + 2\sqrt{5} + 5}{2^2 - (\sqrt{5})^2} \\ &= \frac{4 + 4\sqrt{5} + 5}{4 - 5} \\ &= \frac{9 + 4\sqrt{5}}{-1} \\ &= -9 - 4\sqrt{5} \end{aligned}$$

Example #2 – rationalise

$$\frac{2}{6+3\sqrt{5}}$$

multiply top and bottom by $6-3\sqrt{5}$

$$\begin{aligned} & \frac{2(6-3\sqrt{5})}{(6+3\sqrt{5})(6-3\sqrt{5})} \\ &= \frac{12-6\sqrt{5}}{6^2-(3\sqrt{5})^2} \\ &= \frac{12-6\sqrt{5}}{6^2-(3\sqrt{5})^2} = \frac{12-6\sqrt{5}}{6^2-(3\times 3\times \sqrt{5}\times \sqrt{5})} \\ &= \frac{12-6\sqrt{5}}{6^2-(9\times 5)} = \frac{12-6\sqrt{5}}{36-45} = \frac{12-6\sqrt{5}}{-9} = -\frac{(12-6\sqrt{5})}{9} \\ &= -\frac{3(4-2\sqrt{5})}{9} = -\frac{(4-2\sqrt{5})}{3} = \frac{-4+2\sqrt{5}}{3} \end{aligned}$$

Reduction of Surds - This is a way of making the square root smaller by examining its squared factors and removing them.

$$\sqrt{18} = \sqrt{(3\times 3)\times 2} = \sqrt{3\times 3}\times \sqrt{2} = 3\sqrt{2}$$

$$\sqrt{48} = \sqrt{(2\times 2)\times (2\times 2)\times 3} = 2\times 2\sqrt{3} = 4\sqrt{3}$$

$$\sqrt{63} = \sqrt{(3\times 3)\times 7} = 3\sqrt{7}$$

Rational and Irrational Numbers - In the test for rational and irrational numbers, if a surd has a square root in the numerator, while the denominator is '1' or some other number, then the number represented by the expression is 'irrational'.

examples of irrational surds:

$$\frac{3+\sqrt{3}}{2}, \quad \frac{5+\sqrt{7}}{3}, \quad \frac{6-\sqrt{2}}{5}$$