

Algebra : Inequalities

Symbols

$x > y$	x greater than y
$x < y$	x less than y
$x \geq y$	x greater than or equal to y
$x \leq y$	x less than or equal to y

The rules of inequalities (sometimes called 'inequations')

These are the same as for equations i.e that whatever you do to one side of the equation (add/subtract, multiply/divide by quantities) you must do to the other.

However, there are **two** exceptions to these rules.

When you **multiply each side by a negative quantity**

'<' becomes '>', or '>' becomes '<' .

That is, the **inequality sign is reversed**.

Similarly, when you **divide each side by a negative quantity**

< becomes >, or > becomes < .

That is, the **inequality sign is reversed**.

Examples

$$-\frac{x}{2} < 6$$

multiplying each side by -2

$$\frac{-2}{-2}x > -12 \text{ (note } < \text{ to } > \text{)}$$

$$\underline{x > -12}$$

$$-5x > 4$$

dividing each side by -5

$$\frac{-5}{-5}x < \frac{4}{-5} \text{ (note } > \text{ to } < \text{)}$$

$$x < \frac{4}{-5}$$

$$\underline{x < -\frac{4}{5}}$$

Inequalities with one variable

Example #1 - Find all the integral values of x where,

$$6 \geq x > -5$$

The values of x lie equal to and less than 6 but greater than -5, but not equal to it.

The integral (whole numbers + or - or zero) values of x are therefore:

$$\underline{6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4}$$

Example #2 - What is the range of values of x where,

$$x^2 \geq 144$$

Since the square root of 144 is +12 or -12 (remember two negatives multiplied make a positive), x can have values between 12 and -12.

In other words the value of x is less than or equal to 12 and more than or equal to -12. This is written:

$$\underline{12 \geq x \geq -12}$$

Inequalities with two variables - Solution is by arranging the equation into the form

$$Ax + By = C$$

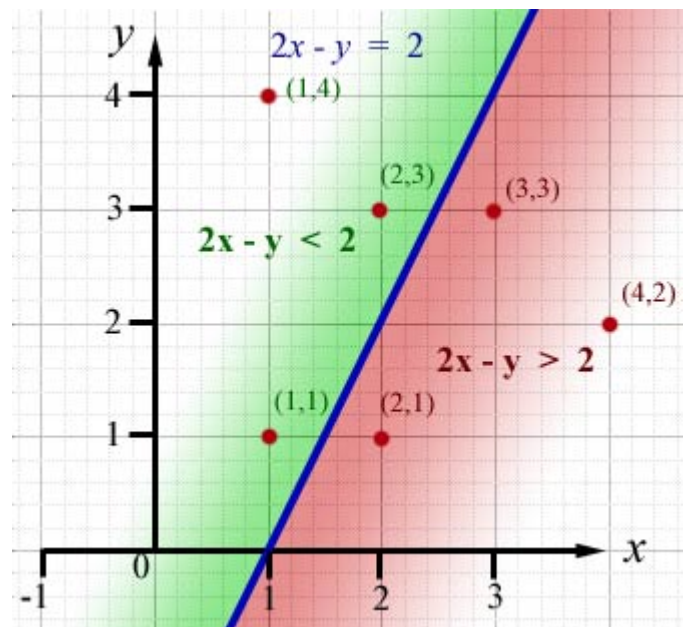
Then, above the line of the equation, $Ax + By < C$

and below the line, $Ax + By > C$

Consider the graph of $-2x + y = -2$

note - the first term **A** must be made positive by multiplying the whole equation by -1

The equation $-2x + y = -2$ becomes $2x - y = 2$



look at the points(red) and the value of $2x - y$ for each. The table below summarises the result.

point(x,y)	$2x - y$	value	more than 2 ?	above/below
(1,1)	$2(1)-1(1)$	1	no - less	above
(1,4)	$2(1)-(4)$	-2	no - less	above
(2,3)	$2(2)-(3)$	1	no - less	above
(3,3)	$2(3)-(3)$	3	yes - more	below
(2,1)	$2(2)-(1)$	3	yes - more	below
(4,2)	$2(4)-(2)$	6	yes - more	below

The Modulus

The **modulus** is the numerical value of a number, irrespective of the sign it carries.

hence $|x| < 3$ means $-3 < x < 3$

Example

$$|2x+1| < 5$$

$$\Rightarrow -5 < 2x+1 < 5$$

$$-5 < 2x+1$$

$$-5-1 < 2x$$

$$-6 < 2x$$

$$\underline{-3 < x}$$

$$2x+1 < 5$$

$$2x < 5-1$$

$$2x < 4$$

$$\underline{x < 2}$$

$$\underline{-3 < x < 2}$$