Algebra : Indices

The Laws of Indices

$$p^m \times p^n = p^{m+n}$$

$$\frac{p^m}{p^n} = p^{m-n}$$

$$(p^m)^n = p^{m \times n} = p^{mn}$$

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

$$\frac{1}{p^m} = p^{-m} \qquad p^n = \frac{1}{p^{-n}}$$

$$p^0 = 1$$

Indices - Multiplication

remembering that:

$$p^m \times p^n = p^{m+n}$$

Examples

$$a^2 \times a^5 = a^7$$
$$a^{-2}b^3 \times a^5 b^{-4} = a^3 b^{-1}$$
$$a^3 b^2 \times a^4 b^7 = a^7 b^9$$

$$2a^3b^2c^{-3} \times 5a^3b^{-2}c^2 = 10a^6b^0c^{-1} = 10a^6c^{-1} \qquad (b^0 = 1)$$

$$5a^2b^{-7}c^{-2} \times 6a^{-2}b^5c^3 = 30a^0b^{-2}c^1 = 30b^{-2}c \qquad (a^0 = 1,\ c^1 = c)$$

Indices - Division

remembering that:

$$\frac{p^m}{p^n} = p^{m-n}$$

Examples:

$$\frac{a^5}{a^6} = a^{-1}$$

$$\frac{a^6b^3}{a^7b^{-5}} = a^{-1}b^{3-(-5)} = a^{-1}b^8$$

$$\frac{a^3b^5}{a^2b^7} = a^1b^{-2} = ab^{-2}$$

$$\frac{12a^3b^2}{3a^{-4}b^4} = 4a^{3-(-4)}b^{-2} = 4a^7b^{-2}$$

$$\frac{8a^4b^2c^{-7}}{2a^3b^{-4}c^{-5}} = 4a^1b^{2-(-4)}c^{-7-(-5)} = 4ab^6c^{-2}$$

Indices - Powers

remembering that:

$$(p^m)^n = p^{m\times n} = p^{mn}$$

Examples:

$(a^3b^5)^3 = a^9b^{15}$ $(a^4b^2)^{-5} = a^{-20}b^{-10}$

$(a^{-2}b^4)^{-3} = a^6b^{-12}$ $(4ab^3)^2 = 16a^2b^6$

$(3a^2b)^3 = 27a^6b^3$ $(2a^3b^4)^3 = 8a^9b^{12}$

$4(2ab^2)^3 = 4(8a^3b^6) = 32a^3b^6$

$3(4a^4b^3)^2 = 3(16a^8b^6) = 48a^8b^6$

Indices - Roots and Reciprocals

remembering that:

$$\sqrt[n]{p} = p^{\frac{1}{n}}$$

and

$$\frac{1}{p^m} = p^{-m} \qquad p^n = \frac{1}{p^{-n}}$$

Examples:

$$\frac{a^3}{b^4} = a^3 b^{-4}$$

$$\frac{a^2}{b^{-3}c^2} = a^2 b^3 c^{-2}$$

$$\frac{ab^2c^3}{b^{-5}c^2} = ab^7 c$$

$$\sqrt[3]{\frac{b^4}{a^{-7}}} = \left(a^7 b^4\right)^{1/3} = a^{7/3} b^{4/3}$$

$$\sqrt{\frac{a^2}{b^6}} = \left(a^2 b^{-6}\right)^{1/2} = a^{2/2} b^{-6/2} = ab^{-3}$$

$$\frac{a^2 b^{1/3}}{a^{1/2} b^3} = a^2 \; a^{-1/2} \; b^{-3} \; b^{1/3}$$

$$= a^{4/2} \; a^{-1/2} \; b^{-9/3} \; b^{1/3} = a^{3/2} \; b^{-8/3}$$