

The General Binomial Expansion ($n \geq 1$)

This is a way of finding all the terms of the series, the coefficients and the powers of the variables.

The coefficients, represented by ${}^n C_r$, are calculated using probability theory. For a deeper understanding you may wish to look at where ${}^n C_r$ comes from; but for now you must accept that:

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

where 'n' is the power/index of the original expression and 'r' is the number order of the term minus one

If n is a positive integer, then:

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 \dots + {}^n C_r a^{n-r} b^r + \dots b^n$$

Example #1

$$(a+b)^5 = a^5 + {}^5 C_1 a^4 b + {}^5 C_2 a^3 b^2 + {}^5 C_3 a^2 b^3 + {}^5 C_4 a b^4 + b^5$$

$${}^5 C_1 = \frac{5!}{(5-1)!1!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)1} = 5$$

$${}^5 C_2 = \frac{5!}{(5-2)!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)2 \cdot 1} = \frac{5 \cdot 4}{2 \cdot 1} = \frac{20}{2} = 10$$

$${}^5 C_3 = \frac{5!}{(5-3)!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)3 \cdot 2 \cdot 1} = \frac{5 \cdot 4}{2 \cdot 1} = \frac{20}{2} = 10$$

$${}^5 C_4 = \frac{5!}{(5-4)!4!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(1)4 \cdot 3 \cdot 2 \cdot 1} = 5$$

$$(a+b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$$

Example #2

write down $(3x - 2)^5$ as a binomial expansion

using

$$(a + b)^5 = a^5 + {}^5C_1 a^4 b + {}^5C_2 a^3 b^2 + {}^5C_3 a^2 b^3 \\ + {}^5C_4 a b^4 + b^5$$

and the values of the coefficients from previous

let $a = 3x$, $b = -2$

$$(3x - 2)^5 = (3x)^5 + {}^5C_1 (3x)^4 (-2) + {}^5C_2 (3x)^3 (-2)^2 \\ + {}^5C_3 (3x)^2 (-2)^3 + {}^5C_4 (3x) (-2)^4 + (-2)^5$$

$$(3x - 2)^5 = 243x^5 + 5(81x^4)(-2) + 10(27x^3)(4) \\ + 10(9x^2)(-8) + 5(3x)(16) + (-32)$$

$$(3x - 2)^5 \\ = 243x^5 - 810x^4 + 1080x^3 + 720x^2 + 240x - 32$$

It is suggested that the reader try making similar questions, working through the calculations and checking the answer [here](#) (max. value of n=8)

The Particular Binomial Expansion

This is for $(1+x)^n$, where n can take any value positive or negative, and x is a fraction ($-1 < x < 1$).

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

Example

Find the first 4 terms of the expression $(x+3)^{1/2}$.

making $(x+3)^{1/2}$ in the form of $(1+x)^n$

$$(x+3)^{1/2} = (3+x)^{1/2} = \left\{3\left(1+\frac{x}{3}\right)\right\}^{1/2}$$

$$= \sqrt{3} \left[1 + \frac{\left(\frac{1}{2}\right)\left(\frac{x}{3}\right)}{1} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{x}{3}\right)^2}{2 \cdot 1} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{3}\right)^3}{3 \cdot 2 \cdot 1} \right]$$

$$= \sqrt{3} \left[1 + \frac{x}{6} - \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{x^2}{9} + \frac{3}{8} \cdot \frac{1}{6} \cdot \frac{x^3}{27} + \dots \right]$$

$$= \sqrt{3} \left[1 + \frac{x}{6} - \frac{x^2}{72} + \frac{3x^3}{1296} + \dots \right]$$

$$\underline{(x+3)^{1/2} = \sqrt{3} + \frac{\sqrt{3}x}{6} - \frac{\sqrt{3}x^2}{72} + \frac{\sqrt{3}x^3}{1296} \dots}$$