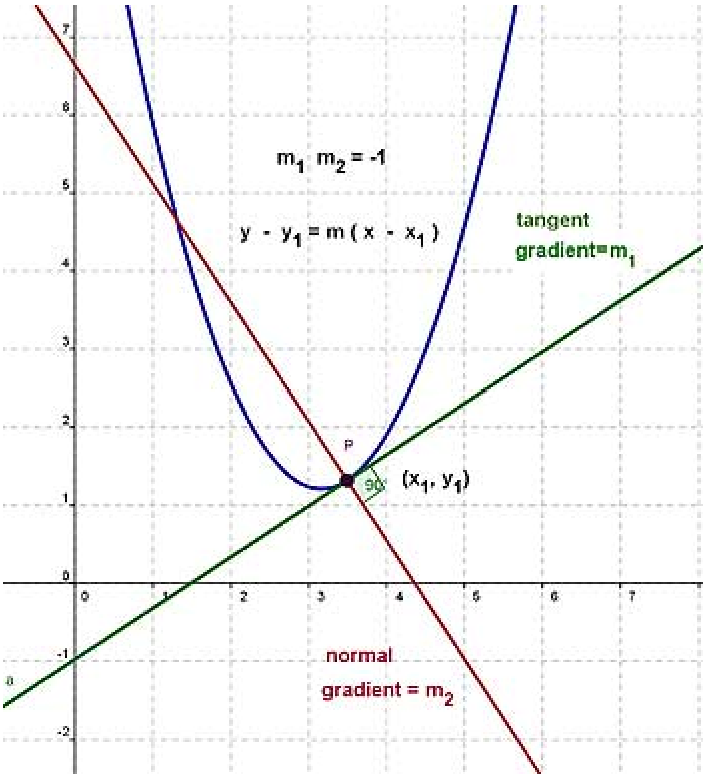
Calculus: Tangents & Normals

Tangents

The gradient of the tangent to the curve $\mathbf{y = f(x)}$ at the point $\mathbf{(x_1, y_1)}$ on the curve is given by:

the value of $\mathbf{dy/dx}$, when $\mathbf{x = x_1}$ and $\mathbf{y = y_1}$



Normals

Two lines of gradients m_1, m_2 respectively are perpendicular to eachother if the product,

$$\mathbf{m_1 \times m_2 = -1}$$

Equation of a tangent

The equation of a tangent is found using the equation for a straight line of gradient **m**, passing through the point **(x_1, y_1)**

$$y - y_1 = m(x - x_1)$$

To obtain the equation we substitute in the values for x_1 and y_1 and **m** (dy/dx) and rearrange to make y the subject.

Example

Find the equation of the tangent to the curve y = $2x^2$ at the point (1,2).

$$y = 2x^2$$

therefore gradient, $\frac{dy}{dx} = 4x$

when $x = 1$ $\frac{dy}{dx} = 4$ therefore gradient is 4

using $y - y_1 = m(x - x_1)$ *

*from coordinate geometry, gradient m of the line between two points (x,y) and (x_1, y_1)

$x_1 = 1$ $y_1 = 2$

$$y - 2 = 4(x - 1)$$
$$y - 2 = 4x - 4$$
$$y = 4x - 4 + 2$$
$$\underline{y = 4x - 2}$$

Equation of a normal

The equation of a normal is found in the same way as the tangent. The gradient(**m_2**)of the normal is calculated from;

m_1 x **m_2 = -1** (where **m_1** is the gradient of the tangent)

so

m_2 = - 1/(**m_1**)

Example

Find the equation of the normal to the curve:

$y = x^2 + 4x + 3$, at the point (-1,0).

$$y = x^2 + 4x + 3$$

therefore gradient(m_1), $\dfrac{dy}{dx} = 2x + 4$

at the point (-1,0) $m_1 = 2x + 4 = -2 + 4 = 2$

let the gradient of the normal be m_2

product of tangent and normal gradient:

$$m_1.m_2 = -1$$

$m_1 = 2$

$\therefore \quad 2.m_2 = -1 \qquad m_2 = -\frac{1}{2}$

using $\quad y - y_1 = m_2(x - x_1)$

when $\quad x_1 = -1, \qquad y_1 = 0$

$$y - 0 = -\tfrac{1}{2}(x - (-1))$$

$$y = -\tfrac{1}{2}x - \tfrac{1}{2}$$

mult. by 2 $\quad 2y = -x - 1$

$\underline{2y + x + 1 = 0}$