6665
Edexcel GCE

Pure Mathematics C3

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI-89, TI-92, Casio *cfx* 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics C3), the paper reference (6665), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Edexcel
Success through qualifications

1. The function f is defined by

$$f: x \mapsto |x - 2| - 3, x \in \mathbb{R}.$$

(a) Solve the equation $f(x) = 1$.

(3)

The function g is defined by

$$g: x \mapsto x^2 - 4x + 11, x \geq 0.$$

(b) Find the range of g.

(3)

(c) Find gf(–1).

(2)

2.
$$f(x) = x^3 - 2x - 5.$$

(a) Show that there is a root α of $f(x) = 0$ for x in the interval [2, 3].

(2)

The root α is to be estimated using the iterative formula

$$x_{n+1} = \sqrt{\left(2 + \frac{5}{x_n}\right)}, \quad x_0 = 2.$$

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures.

(3)

(c) Prove that, to 5 significant figures, α is 2.0946.

(3)

---

3. (a) Using the identity for cos $(A + B)$, prove that $\cos\theta \equiv 1 - 2\sin^2(\tfrac{1}{2}\theta)$.

(3)

(b) Prove that $1 + \sin\theta - \cos\theta \equiv 2\sin(\tfrac{1}{2}\theta)[\cos(\tfrac{1}{2}\theta) + \sin(\tfrac{1}{2}\theta)]$.

(3)

(c) Hence, or otherwise, solve the equation

$$1 + \sin\theta - \cos\theta = 0, \quad 0 \leq \theta < 2\pi.$$

(4)

---

4. $$f(x) = x + \frac{3}{x-1} - \frac{12}{x^2+2x-3}, \; x \in \mathbb{R}, x > 1.$$

(a) Show that $f(x) = \dfrac{x^2+3x+3}{x+3}$.

(5)

(b) Solve the equation $f'(x) = \frac{22}{25}$.

(5)

---

5. **Figure 1**

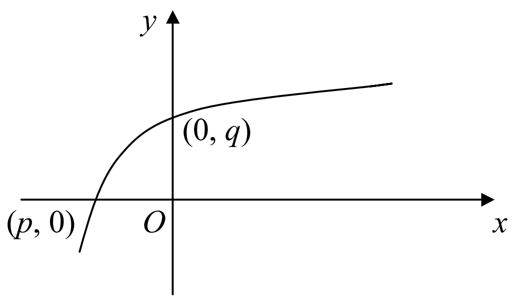

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$. The curve meets the x-axis at P $(p, 0)$ and meets the y-axis at Q $(0, q)$.

(a) On separate diagrams, sketch the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 3f(\frac{1}{2}x)$.

In each case show, in terms of p or q, the coordinates of points at which the curve meets the axes.

(5)

Given that $f(x) = 3\ln(2x + 3)$,

(b) state the exact value of q,

(1)

(c) find the value of p,

(2)

(d) find an equation for the tangent to the curve at P.

(4)

---

6. As a substance cools its temperature, T °C, is related to the time (t minutes) for which it has been cooling. The relationship is given by the equation

$$T = 20 + 60e^{-0.1t}, \quad t \geq 0.$$

(a) Find the value of T when the substance started to cool.

(1)

(b) Explain why the temperature of the substance is always above 20°C.

(1)

(c) Sketch the graph of T against t.

(2)

(d) Find the value, to 2 significant figures, of t at the instant $T = 60$.

(4)

(e) Find $\dfrac{dT}{dt}$.

(2)

(f) Hence find the value of T at which the temperature is decreasing at a rate of 1.8 °C per minute.

(3)

---

7. (i) Given that $y = \tan x + 2\cos x$, find the exact value of $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$.

(3)

(ii) Given that $x = \tan \frac{1}{2}y$, prove that $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

(4)

(iii) Given that $y = e^{-x}\sin 2x$, show that $\dfrac{dy}{dx}$ can be expressed in the form $R\,e^{-x}\cos(2x + \alpha)$. Find, to 3 significant figures, the values of R and α, where $0 < \alpha < \dfrac{\pi}{2}$.

(7)

END

Turn over

Paper Reference(s)

6666

Edexcel GCE

Pure Mathematics C4

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI-89, TI-92, Casio CFX-9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics C4), the paper reference (6666), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Use the binomial theorem to expand $(4-3x)^{-\frac{1}{2}}$, in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

---

2. The curve C has equation

$$13x^2 + 13y^2 - 10xy = 52.$$

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x and y, simplifying your answer.

(6)

---

3. Use the substitution $x = \tan\theta$ to show that

$$\int_0^1 \frac{1}{(1+x^2)^2}\,\mathrm{d}x = \frac{\pi}{8} + \frac{1}{4}\,.$$

(8)

---

4. **Figure 1**

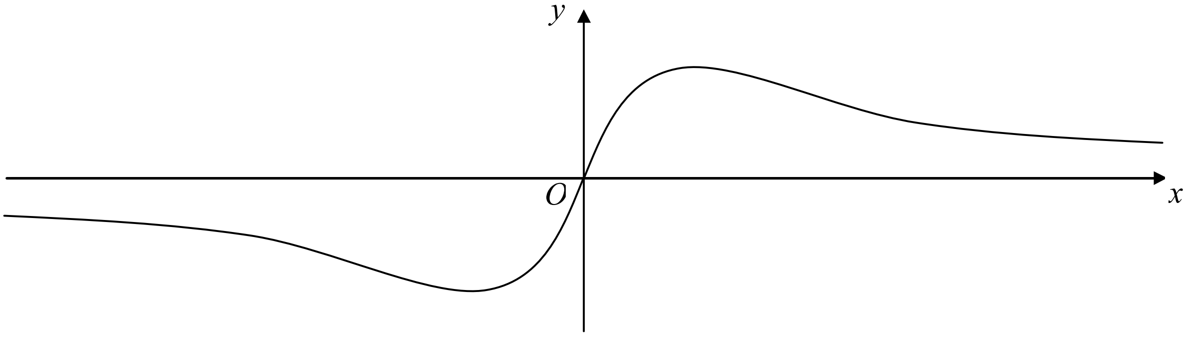

Figure 1 shows part of the curve with parametric equations

$$x = \tan t, \quad y = \sin 2t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find the gradient of the curve at the point P where $t = \frac{\pi}{3}$.

(4)

(b) Find an equation of the normal to the curve at P.

(3)

(c) Find an equation of the normal to the curve at the point Q where $t = \frac{\pi}{4}$.

(2)

5. The vector equations of two straight lines are

$$\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \quad \text{and}$$

$$\mathbf{r} = 2\mathbf{i} - 11\mathbf{j} + a\mathbf{k} + \mu(-3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}).$$

Given that the two lines intersect, find

(a) the coordinates of the point of intersection,

(5)

(b) the value of the constant a,

(2)

(c) the acute angle between the two lines.

(4)

---

6. Given that

$$\frac{11x-1}{(1-x)^2(2+3x)} \equiv \frac{A}{(1-x)^2} + \frac{B}{(1-x)} + \frac{C}{(2+3x)},$$

(a) find the values of A, B and C.

(4)

(b) Find the exact value of $\int_0^{\frac{1}{2}} \frac{11x-1}{(1-x)^2(2+3x)}\,\mathrm{d}x$, giving your answer in the form $k + \ln a$, where k is an integer and a is a simplified fraction.

(7)

---

7. (a) Given that $u = \frac{x}{2} - \frac{1}{8}\sin 4x$, show that $\frac{\mathrm{d}u}{\mathrm{d}x} = \sin^2 2x$.

(4)

Figure 2

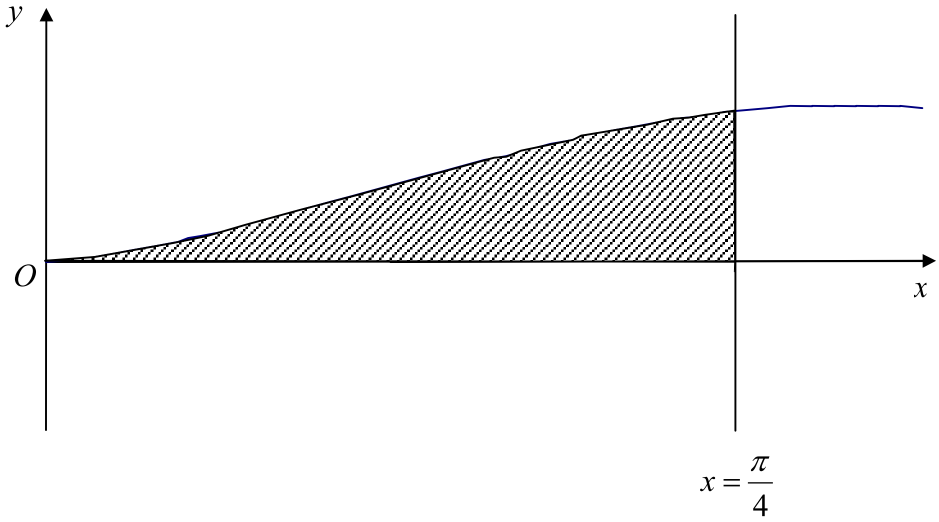

Figure 2 shows the finite region bounded by the curve $y = x^{\frac{1}{2}} \sin 2x$, the line $x = \frac{\pi}{4}$ and the x-axis. This region is rotated through 2π radians about the x-axis.

(b) Using the result in part (a), or otherwise, find the exact value of the volume generated.

(8)

8. A circular stain grows in such a way that the rate of increase of its radius is inversely proportional to the square of the radius. Given that the area of the stain at time t seconds is A cm^2,

(a) show that $\frac{\mathrm{d}A}{\mathrm{d}t} \propto \frac{1}{\sqrt{A}}$.

(6)

Another stain, which is growing more quickly, has area S cm² at time t seconds. It is given that

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{2\mathrm{e}^{2t}}{\sqrt{S}}.$$

Given that, for this second stain, $S = 9$ at time $t = 0$,

(b) solve the differential equation to find the time at which $S = 16$. Give your answer to 2 significant figures.

(7)

END